Team Apache Stealth: Unscented Kalman Filter for Attitude Estimation

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***Abstract*—This report aims to explain attitude estimation techniques for orientation determination using data from a 6-DoF Inertial Measurement Unit, comprising a 3-DoF gyroscope and a 3-DoF accelerometer. We explore three primary approaches: Complementary filter-based estimation, Madgwick filter-based estimation, and Unscented Kalman filter-based estimation. We generate plots for each method and conduct a comparative analysis to assess their strengths and weaknesses. To enhance the validity of our findings, we introduce Vicon data as a benchmark, serving as the ground truth for evaluating the accuracy of attitude estimates produced by the different approaches. Our analysis indicates that the Unscented Kalman filter performs the best among the methods studied.**

I. Introduction

In aerial robotics applications, the precise determination of the system's orientation holds great importance in providing feedback to autopilot systems and controllers. This task is often performed using the Inertial Measurement Unit(IMU), which is equipped with tri-axis gyroscopes and accelerometers. Gyroscopes measure angular velocity, allowing for integration over time to approximate orientation. However, this numerical integration is susceptible to accumulating errors, which causes divergence from the true attitude. On the other hand, accelerometers gauge Earth's gravitational field, furnishing orientation estimates within an absolute frame of reference. However, these estimates can be compromised when the system undergoes translation motion.

One approach to address this challenge is through the utilization of filters. These filters combine accelerometer and gyroscope data from the IMU to produce a unified and more reliable orientation estimate. In this context, we will delve into three distinct methods for orientation determination: Complementary Filters, Madgwick Filters, and Unscented Kalman Filters. Each of these methods offers unique approaches and advantages in tackling the critical task of precise orientation estimation in aerial robotics applications.

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II. Data Pre-pocessing

This section outlines the methodologies employed to estimate and rectify biases, as well as scale the raw data values obtained from the IMU into SI units for both the accelerometer and gyroscope sensors.

A. Accelerometer IMU Data

Prior to analysis, the data extracted from the IMU necessitates pre-processing to convert it into meaningful physical units and counteract any inherent instrumentation bias. The subsequent expression explains the conversion process of raw accelerometer readings, denoted as $a = [a_x \; a_y \; a_z]^T$, into acceleration data represented in m/s^2.

$$\hat{a_i} = 9.81((a_i \times s_{a,i}) + b_{a,i})$$

Here $\hat{a}_i$ represents a_i in physical units, b_{a_i} represents bias and s_{a_i} represents the scale factor for i^{th} axis. Where $i \in x, y, z$ axis.

B. Gyroscope IMU Data

Similarly, the data originating from the gyroscopes underwent processing to convert the raw gyro angular velocity reading $\omega = [\omega_x \ \omega_y \ \omega_z]^T$ into angular velocity data in rad/s, the following expression is used.

$$\hat{\omega_i} = \frac{3300}{1023} \times \frac{\pi}{180} \times 0.3 \times (\omega_i - b_{g,i})$$

To compute the bias term, the average of the initial 200 gyroscope measurements is derived. This assumption is based on the stability of the gyroscope during the initial 200 measurements within each dataset, which is utilized to establish the initial angular velocities. For each axis bias term is determined. Where $k = 200$.

$$b_{g,i} = \frac{1}{k} \sum_{i=1}^{k} \omega_i$$

III. Orientation Determination

In this section, we have addressed the methodology for ascertaining orientation from separately preprocessed accelerometer and gyroscope data. By leveraging the strengths of each sensor, we have formulated the implementation of a complementary filter. This filter integrates the favorable aspects of both datasets to generate an enhanced orientation estimation. Finally, the orientation obtained through the implemented methods is compared with the ground truth observations extracted from the Vicon dataset.

A. Orientation from accelerometer

The estimated orientation values were derived from the accelerometer data by determining how the acceleration vector aligns with the familiar orientation of the gravitational vector. This estimation was achieved using the following simple trigonometric relationships.

$$Roll(\phi) = tan^{-1}(a_y/\sqrt{(a_x)^2 + (a_z)^2})$$

$$Pitch(\theta) = tan^{-1}(a_x/\sqrt{(a_y)^2 + (a_z)^2})$$

$$Yaw(\psi) = tan^{-1}(\sqrt{(a_x)^2 + (a_y)^2}/a_z)$$

However, it's crucial to acknowledge that this method of attitude estimation has limitations. Because of the symmetry of the gravity vector around the z-axis, this approach is inherently imprecise for Yaw measurements. Moreover, distinguishing between acceleration stemming from rapid movement and that resulting from gravity is challenging. As a result, this method isn't suitable for offering accurate estimates during swift motions over brief time intervals.

B. Orientation from Gyroscope

The gyroscope values are integrated to obtain the angles. The integration is performed using quaternions due to their inherent advantages in representing orientation changes. Unlike Euler angles, quaternions do not suffer from gimbal lock, which is a phenomenon where a particular orientation configuration limits the range of motion in certain directions. This makes quaternions more robust for continuous rotations. Quaternions comprise of a single real element (represented by the subscript 0) and three imaginary elements (represented by the subscripts 1, 2, and 3). The following expression describes the attitude quaternion.

$$q = [q_0 \ q_1 \ q_2 \ q_3]^T$$

$${q_0}^2 + {q_1}^2 + {q_2}^2 + {q_3}^2 = 1$$

The updated quaternions is calculated as below:

$$\alpha\Delta = |\vec{\omega_k}|\Delta t$$

$$\vec{e}\Delta = \frac{\vec{\omega_k}}{|\vec{\omega_k}|}$$

$$q_k = \left(cos\left(\frac{\alpha\Delta}{2}\right), \vec{e}\Delta sin\left(\frac{\alpha\Delta}{2}\right)\right)$$

To calculate current state quaternion the following equations are used:

$$\alpha = |\vec{\omega_k}|$$

$$\vec{e} = \frac{\vec{\omega_k}}{|\vec{\omega_k}|}$$

$$q_k = \left(cos\left(\frac{\alpha}{2}\right), \vec{e}sin\left(\frac{\alpha}{2}\right)\right)$$

Now, the new state quaternion is described as (k is the current state and k + 1 is the next state),

$$q_{k+1} = q_k q\Delta$$

Then euler angles are calculated from the quaternions as follows:

$$Roll(\phi) = tan^{-1}\left(\frac{2(q_0q_1 + q_2q_3)}{1 - 2(q_1{}^2 + q_2{}^2)}\right)$$

$$Pitch(\theta) = sin^{-1}(2(q_0q_2 - q_3q_1))$$

$$Yaw(\psi) = tan^{-1}\left(\frac{2(q_0q_3 + q_1q_2)}{1 - 2(q_2{}^2 + q_3{}^2)}\right)$$

This method generally performs well, but offers no way to compensate for noise in the IMU readings, therefore the estimates tend to drift and become more inaccurate with time.

IV. Complementry Filter

Following the independent calculation of attitude using accelerometer and gyroscope data, the derived attitudes can be merged to enhance the overall attitude estimation. The complementary filter employs a predetermined weighted average of both individual components to produce a refined attitude estimation. This blending of accelerometer and gyroscope information optimally leverages the strengths of each source to yield a more accurate and stable attitude representation.

$$\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}_{\text{Comp}} = \begin{bmatrix}\alpha & 0 & 0\\ 0 & \beta & 0\\ 0 & 0 & \gamma\end{bmatrix} \cdot \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}_{\text{Acc}} + \begin{bmatrix}1-\alpha & 0 & 0\\ 0 & 1-\beta & 0\\ 0 & 0 & 1-\gamma\end{bmatrix} \cdot \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}_{\text{Gyro}}$$

Here α, β and γ are the mixing parameters. α was chosen as 0.75, β was chosen as 0.75 and γ was chosen as 0.0 . Effectively the gyro measurements are high pass filtered to remove drift and accelerometer measurements are low pass filtered to remove noise.

V. Unscented Kalman Filter

Unscented Kalman Filter Utilizes the non-linearity of the process model state estimation and covariance matrix estimation through numerous sigma points[**?**]. To initialize the similar to the Kalman filter for attitude estimation, you begin by defining key matrices like process noise Q, measurement noise R, and the covariance matrix P. To generate sigma points, which represent the system's uncertainty, you calculate disturbances W. These disturbances W are sampled from a distribution centered around zero. The generated sigma points are then used to predict and update the system's state based on measurements, providing an estimate of the system's attitude in space. For attitude estimation, we have a state vector of 7 states.

$$x = \begin{bmatrix}q_0\\ q_1\\ q_2\\ q_3\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

Since the state has 6 degrees of freedom, filter initialized covariance matrix P as a $6X6$matrix. Then sigma points are calculated using $CholeskyDecomposition$.

$$S = \sqrt{P_{t-1} + Q}$$

.

Here Q is the process model covariance. The disturbance (noise) of the sigma points is calculated using

$$\mathcal{W}_{i,i+2n} = columns(\pm\sqrt{n}S)$$

The above equations result in 12 sigma points disturbances (noise) which lie on the positive and negative side along eigen vector of covariance matrix P. Now sigma points are calculated by adding these disturbances to the current state.

$$\mathcal{X}_i = \begin{bmatrix}q_{t-1}q_{\mathcal{W}_i}\\ \omega_{t-1} + \omega_{\mathcal{W}_i}\end{bmatrix}$$

Where q_{t-1}, ω_{t-1} reefer to the current state and $\omega_{\mathcal{W}_i}$ is the omega part of $\mathcal{W}$ and $q_{\mathcal{W}_i}$ is the quaternion part of $\mathcal{W}$. After computing the sigma points, the next step involves calculating process model points. Each sigma point is passed through the system's process model, resulting in individual state estimates denoted as $\mathcal{Y}_i$. In this context, the process model is responsible for updating the attitude quaternion using the current rotation rates as inputs. The updated sigma points are computed using.

$$\mathcal{Y}_i = A(\mathcal{X}_i, 0) = \begin{bmatrix} q_{t-1} q_{\mathcal{W}} q_{\Delta} \\ \omega_{t-1} + \omega_{\mathcal{W}_i} \end{bmatrix}$$

where q_{Δ} is calculated for time t assuming the angular velocity remains constant during the time interval Δt

$$\omega_t = \omega_{t-1}$$
$$q_{\Delta} = \left[\cos\left(\frac{|\omega_{t-1}|\Delta t}{2}\right), \frac{\omega_{t-1}}{|\omega_{t-1}|}\sin\left(\frac{|\omega_{t-1}|\Delta t}{2}\right)\right]$$

Unlike the standard Kalman filter, where the estimated state and covariance matrix are obtained through a linear process model and the current covariance matrix, in this UKF , the estimated state and covariance are derived from the outcomes of the sigma points. This approach enables the UKF to effectively handle and model the non-linear aspects of the system's dynamics.

The mean of the sigma points cannot be calculated directly by taking average as each sigma point is representing a rotation through quaternions. There for the mean of the sigma points is calculated by iterating a mean attitude error until a mean attitude is converged upon using intrinsic gradient descent method. (Refer Algorithm 1.)

The covariance matrix can be computed with the mean centered disturbances $\mathcal{W}_i^{'}$ now that they are centered around the mean sigma point.

$$P = \frac{1}{2n}\sum \mathcal{W}_i^{'} \mathcal{W}_i^{'T}$$

where $\mathcal{W}_i^{'}$ is the mean centre $\mathcal{W}$.

$$\mathcal{W}_i^{'} = [\mathcal{X}_i - \bar{x}] = \begin{bmatrix} q_i \bar{q}^{-1} \\ \omega_i - \bar{\omega} \end{bmatrix}$$

Data: $\mathcal{Y}$
Result: $\bar{\mathcal{Y}}$
$q_t = q_{\mathcal{X},0}$;
while $N \neq Maxiter\ or\ |e| > Thld$ **do**
  for $\forall_i$ **do** ;
  $\vec{e}_i = q_i \bar{q}t^{-1}$
  end
  compute mean using,

$$\vec{e} = \frac{1}{2n}\sum_{i=1}^{2n} \vec{e}_i$$
$$\bar{q}_{t+1} = e\bar{q}_t$$

end

$$\bar{\omega} = \frac{1}{2n}\sum_{i=1}^{2n} \omega_i$$

Algorithm 1: Intrinsic Gradient Descent

Once the estimate and covariance estimates are computed, the process model propagation is finished. The measurement model then begins by calculating the estimated system measurements $\mathcal{Z}_i$ based on the sigma points. In this context, the gravity frame is represented using quaternions.

$$\mathcal{Z}_i = \begin{bmatrix} \mathcal{Y}_q^{-1} g \mathcal{Y}_q \\ \mathcal{Y}_\omega \end{bmatrix}$$

The mean of these computed quaternions and angular velocities are computed into a mean measurement estimate z_k. For mean computation, quaternions are converted into rotation vectors.

$$\bar{z} = \frac{1}{2n}\sum \mathcal{Z}_i$$

The covariance of the measurement estimates Pzz is also computed from the measurement estimates.

$$P_{zz} = \frac{1}{2n}\sum [\mathcal{Z} - \bar{z}][\mathcal{Z} - \bar{z}]^T$$

the innovation term is given by:

$$v_k = z_k - \bar{z}$$

where z_k is the observations with stacked accelerometer and gyroscope data readings from the IMU sensor. The innovation covariance is calculated using,

$$P_{vv} = P_{zz} + R$$

.

Here R is the measurement model covariance $6X6$ matrix. In order to calculate Kalman gain cross-covariance matrix is calculated as,

$$P_{xz} = \frac{1}{2n} \sum [\mathcal{W}_i{}'][\mathcal{Z} - \bar{z}]^T$$

With the innovation covariance and cross-covariance matrix, Kalman gain K can be calculated as follows.

$$K = P_{xz} P_{vv}^{-1}$$

After the Kalman gain computation state update and state covariance can be calculated as follows: state update:

$$\hat{x}_t = \hat{x}_{t-1} + K_k v_k$$

state Covariance as,

$$P_t = \hat{P}_{t-1} - K_K P_{vv} K_K^T$$

Then all the above steps are repeated for the next state using the calculated state and covariance

VI. Result

Plots depicting the calculated Roll (ϕ), Pitch (θ), and Yaw (ψ) obtained from various methods are generated for each dataset (1 to 6). These methods encompass individual accelerometer and gyroscope readings, complementary filters, and the UKF. In order to evaluate their accuracy, the Vicon measurements are regarded as the established ground truth against which the performance of each method is assessed. The evaluated datasets from 6 to 10 are test datasets and do not contain Vicon measurements

VII. Conclusion

The results demonstrate varying accuracy among different attitude estimation methods, with the UKF filter showing the closest alignment to Vicon truth data.

The Complementary filter is a simple and computationally efficient option that can provide reasonably accurate estimates. It is susceptible to drift over time. The Madgwick filter, on the other hand, strikes a balance between accuracy and computational complexity. Its versatility and ease of implementation make it attractive for orientation estimate tasks. However, the Madgwick filter is primarily designed for simpler orientation estimation tasks and may struggle with highly nonlinear systems. The Unscented Kalman Filter(UKF) is the most sophisticated among these filters and offers the highest potential accuracy. The UKF can potentially provide more precise estimates than the Madgwick filter because the UKF uses an approach that considers the nonlinearity of the system dynamics and sensor measurements in its estimation process. However, it demands more computational resources and is difficult to model conceptually. It should also be noted that it is extremely sensitive to noise matrix Q & R and therefore, fine-tuning is required.

Acknowledgment

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References

[1] E. Kraft, "A quaternion-based unscented Kalman filter for orientation tracking," Sixth International Conference of Information Fusion, 2003. Proceedings of the, Cairns, QLD, Australia, 2003, pp. 47-54, doi: 10.1109/ICIF.2003.177425. **Link**

[2] Orienation Tracking based Panorama Stitching using Unscented Kalman Filter **Link**

1. TRAIN DATASET 1

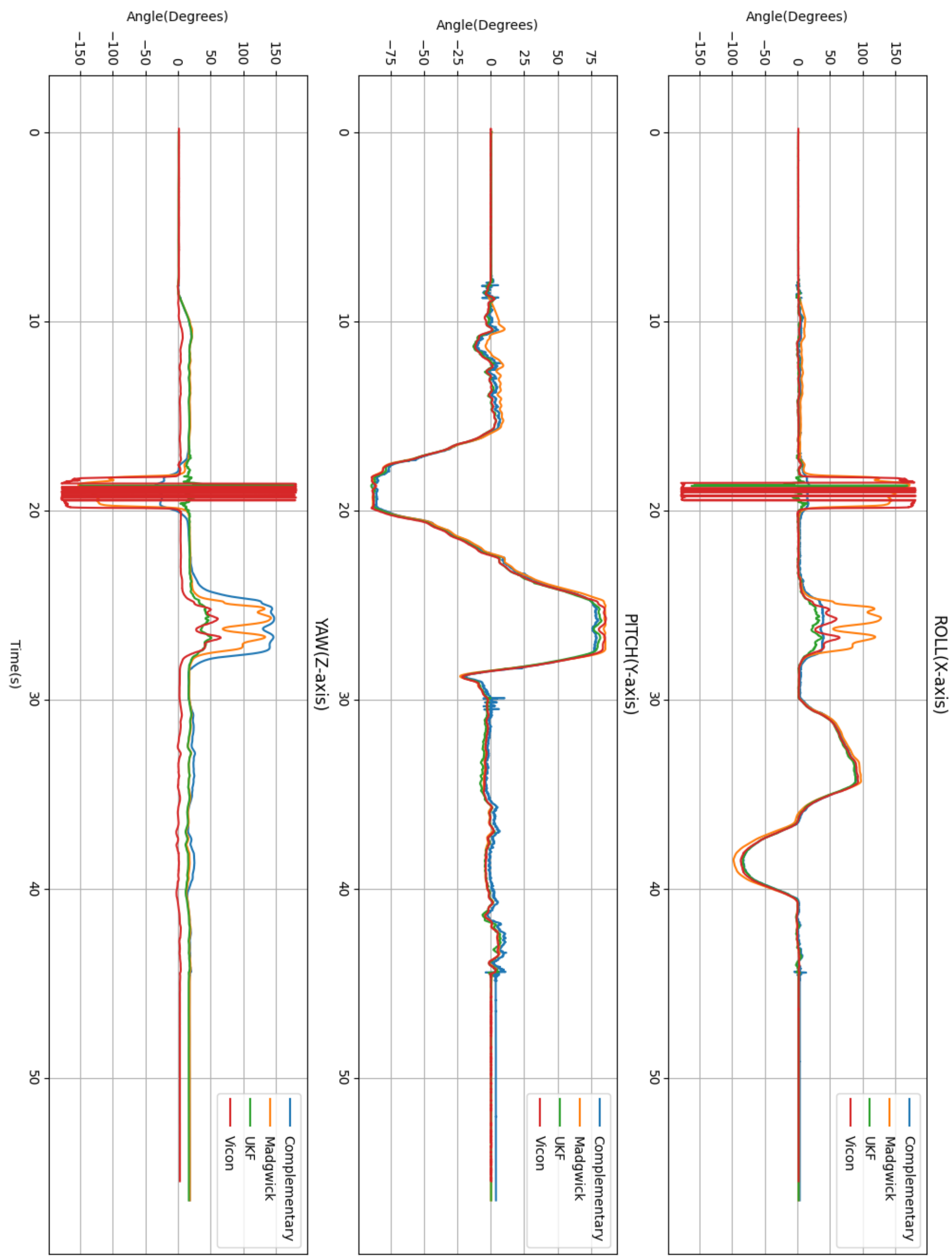

Fig. 1: Comparison of Attitude Estimations for dataset 1

2. TRAIN DATASET 2

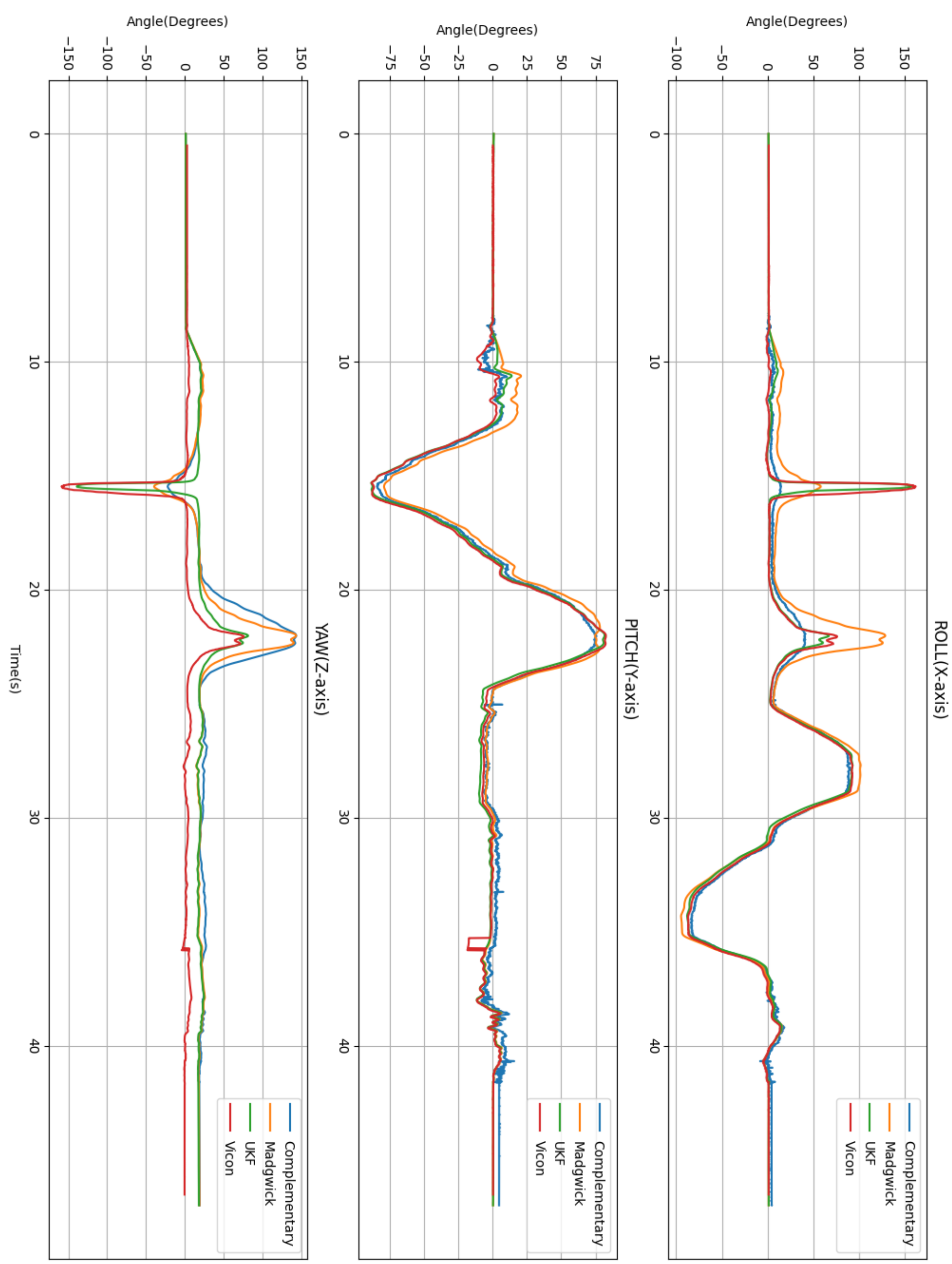

Fig. 2: Comparison of Attitude Estimation for dataset 2

3. TRAIN DATASET 3

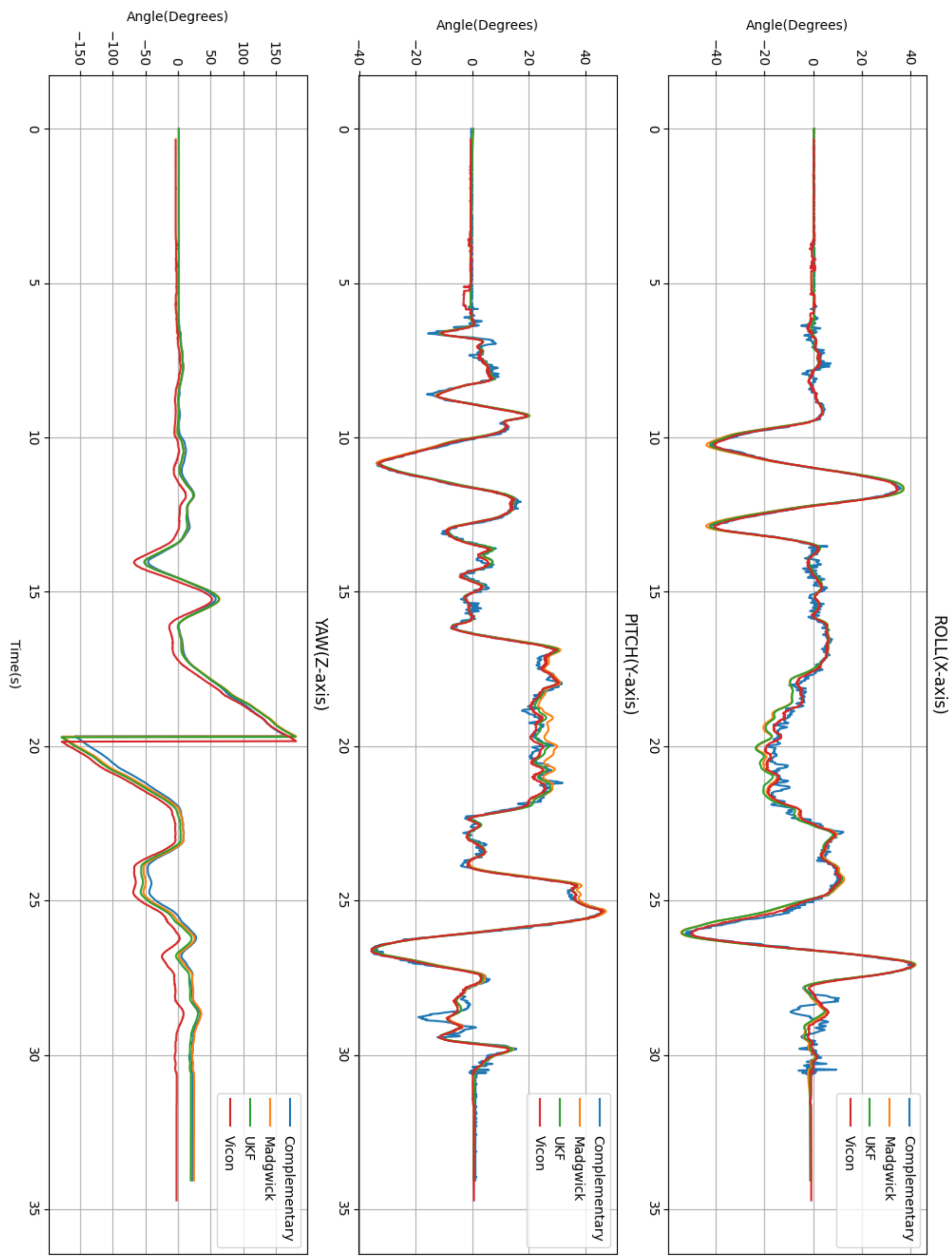

Fig. 3: Comparison of Attitude Estimation for dataset 3

4. TRAIN DATASET 4

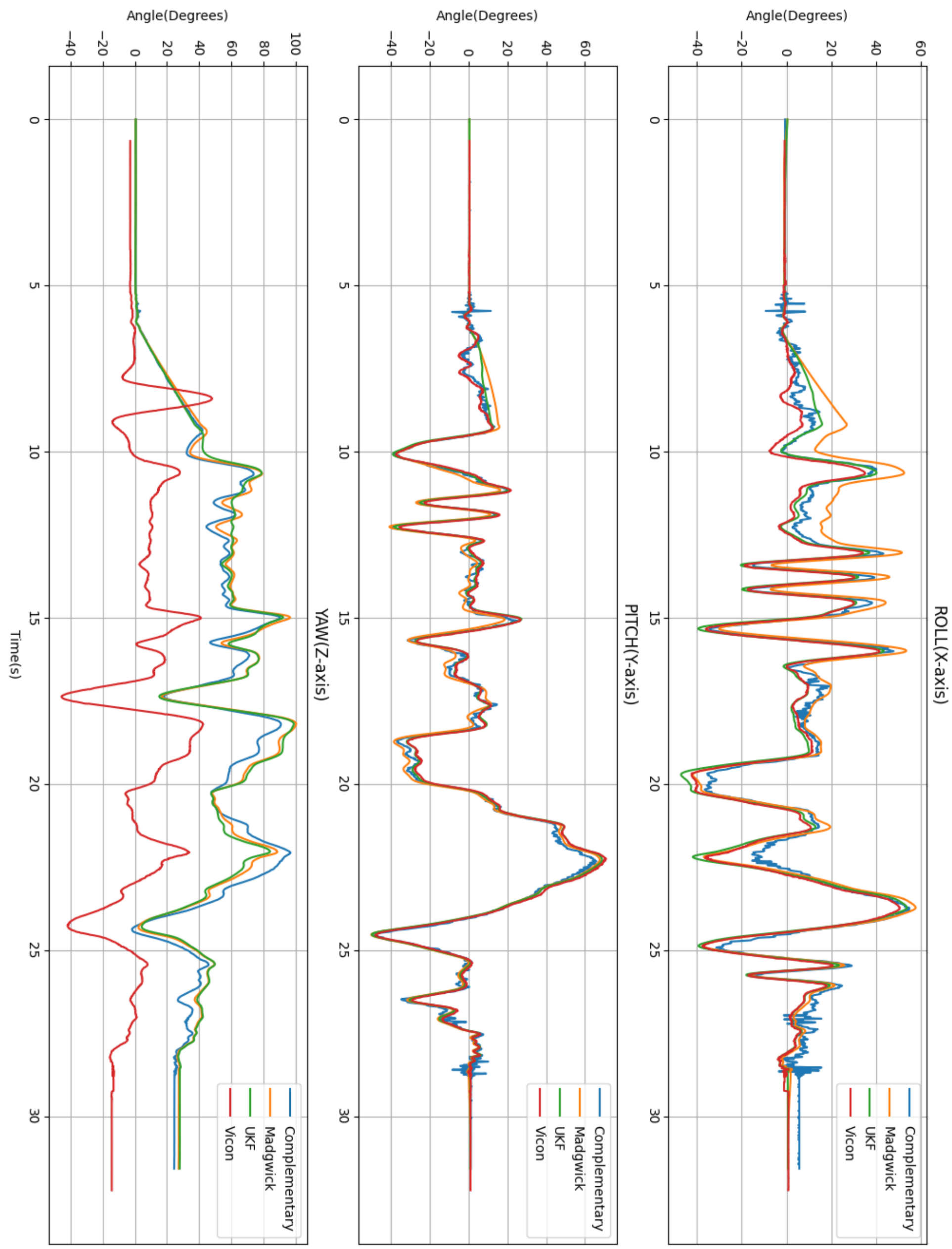

Fig. 4: Comparison of Attitude Estimation for dataset 4

5. TRAIN DATASET 5

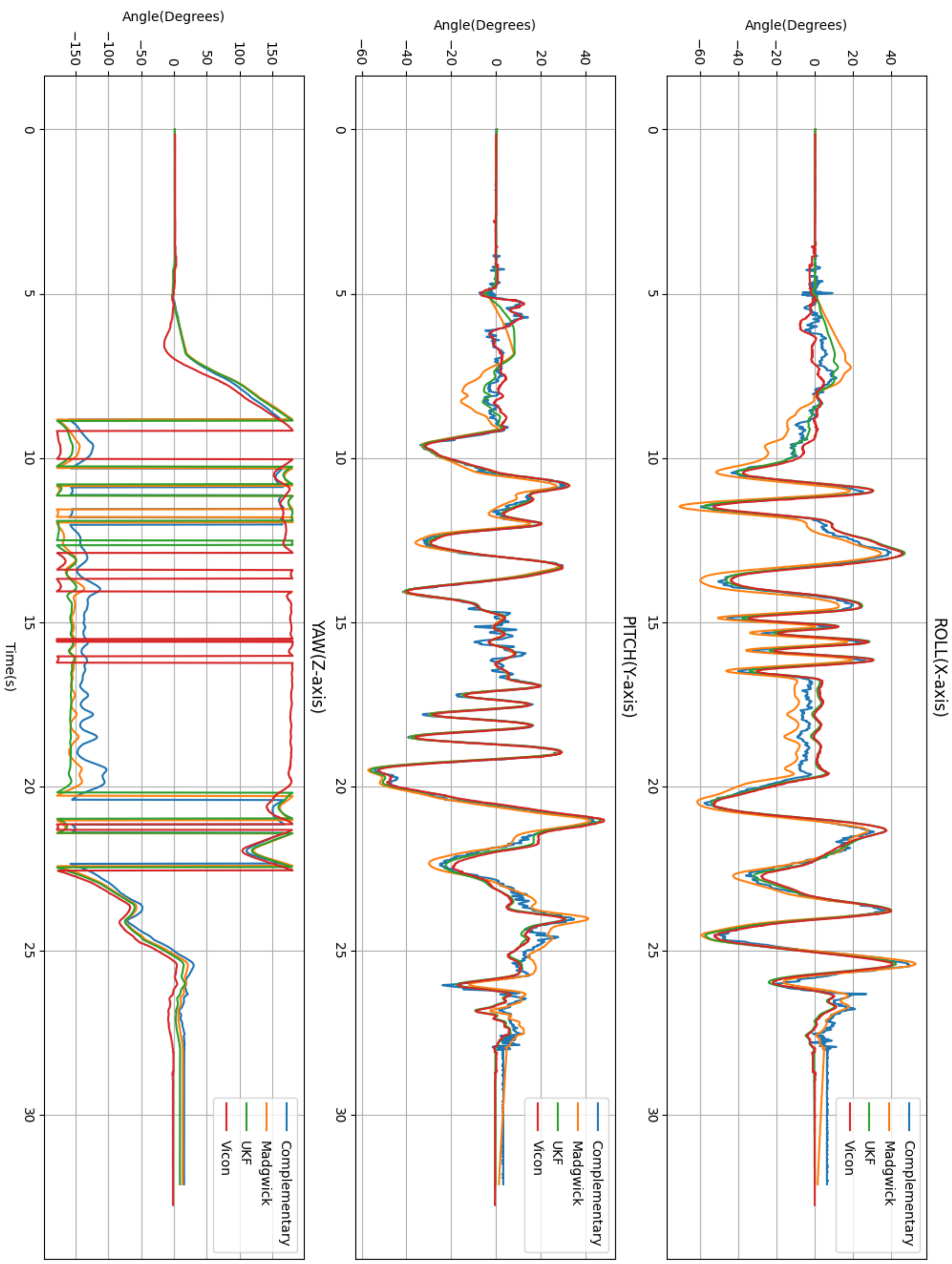

Fig. 5: Comparison of Attitude Estimation for dataset 5

6. TRAIN DATASET 6

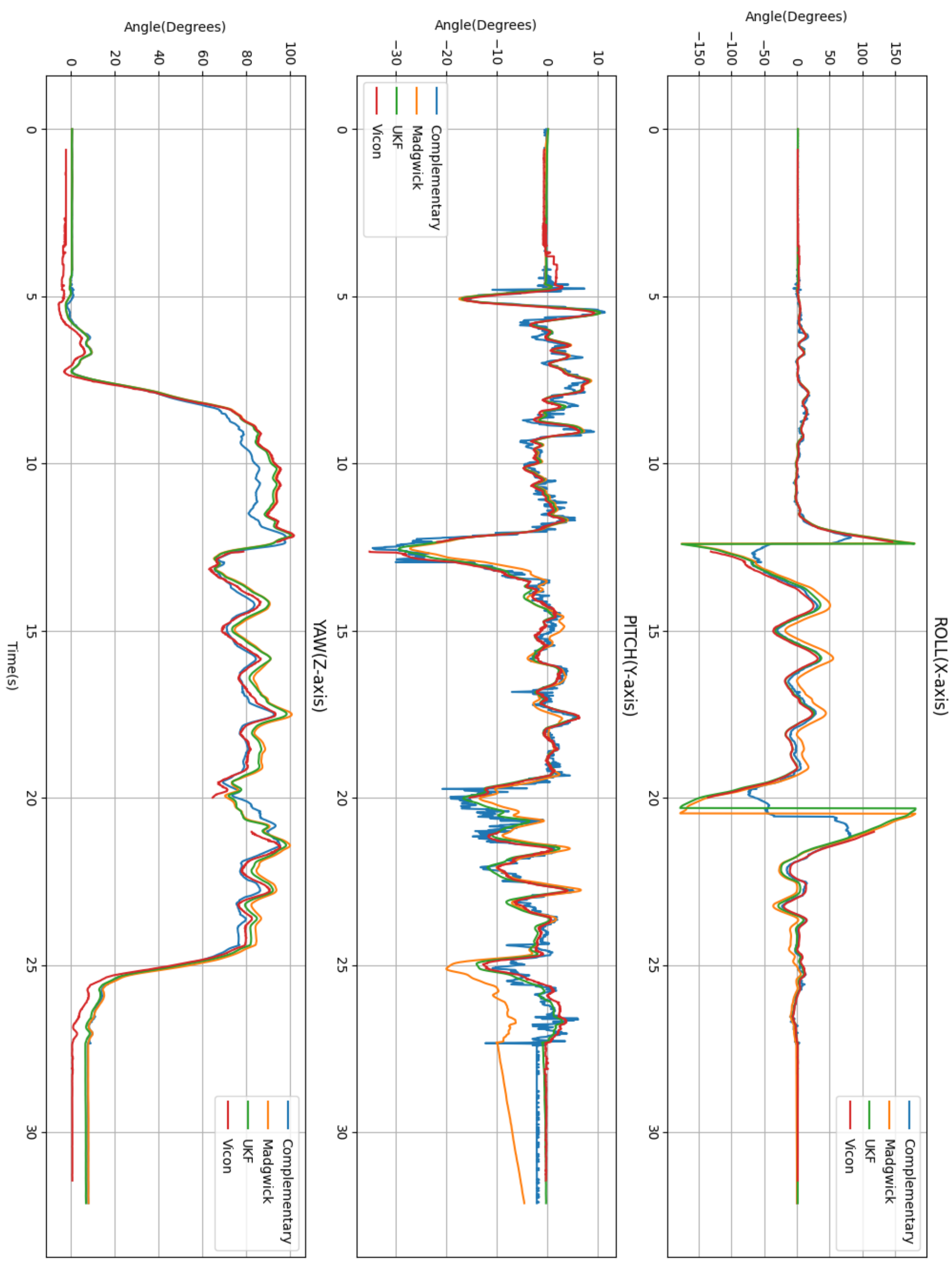

Fig. 6: Comparison of Attitude Estimation for dataset 6

7. TEST DATASET 7

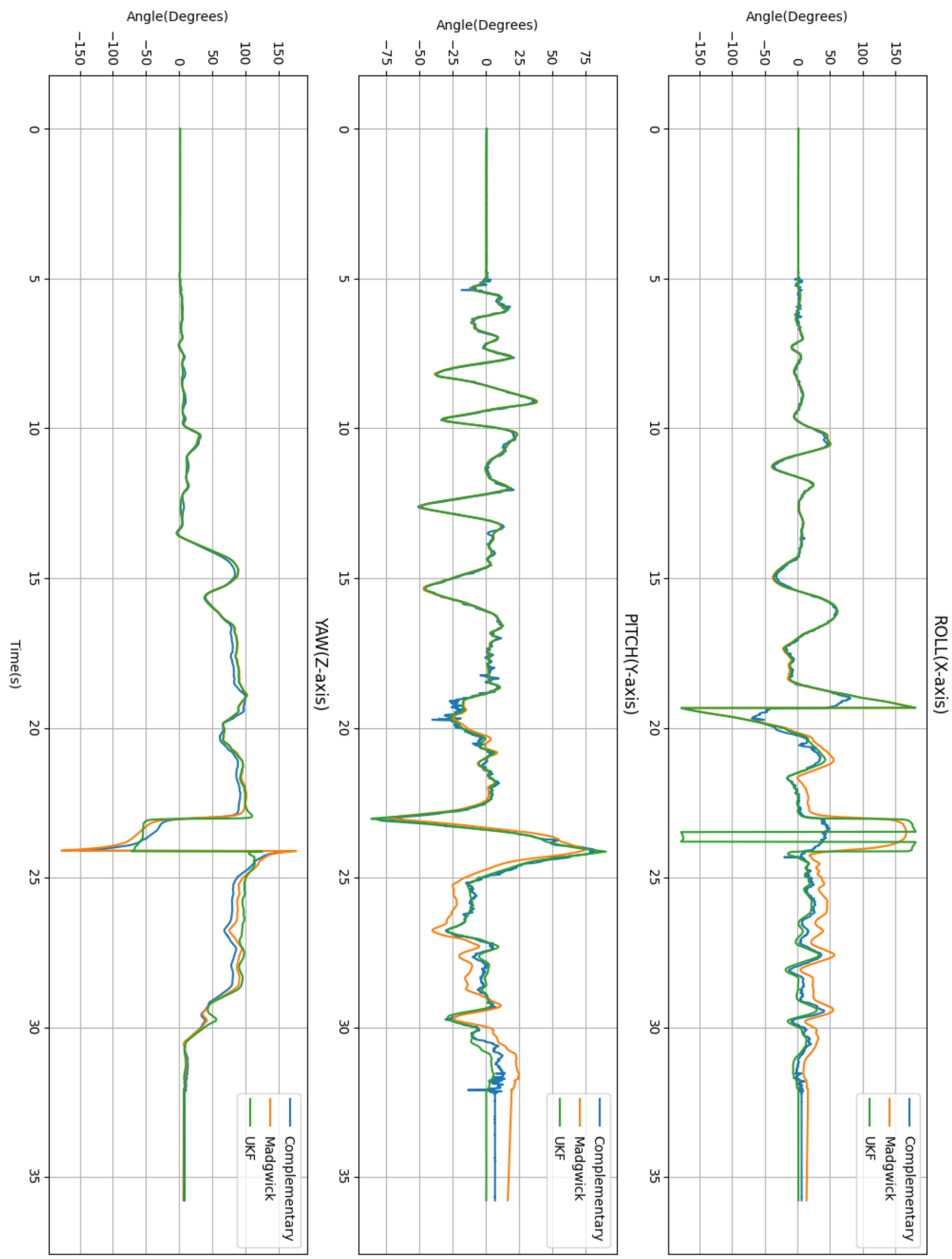

Fig. 7: Comparison of Attitude Estimation for dataset 7

8. TEST DATASET 8

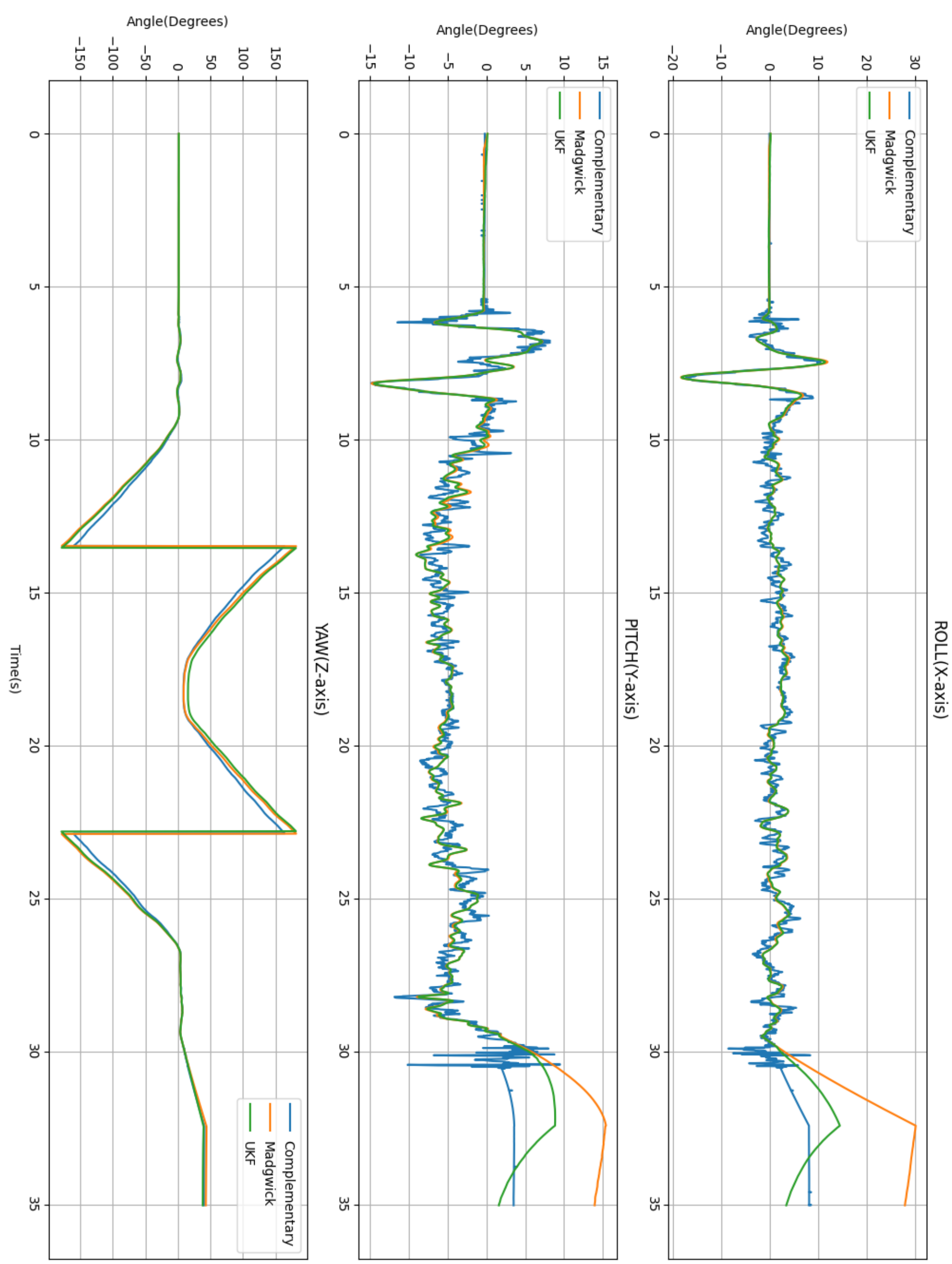

Fig. 8: Comparison of Attitude Estimation for dataset 8

9. TEST DATASET 9

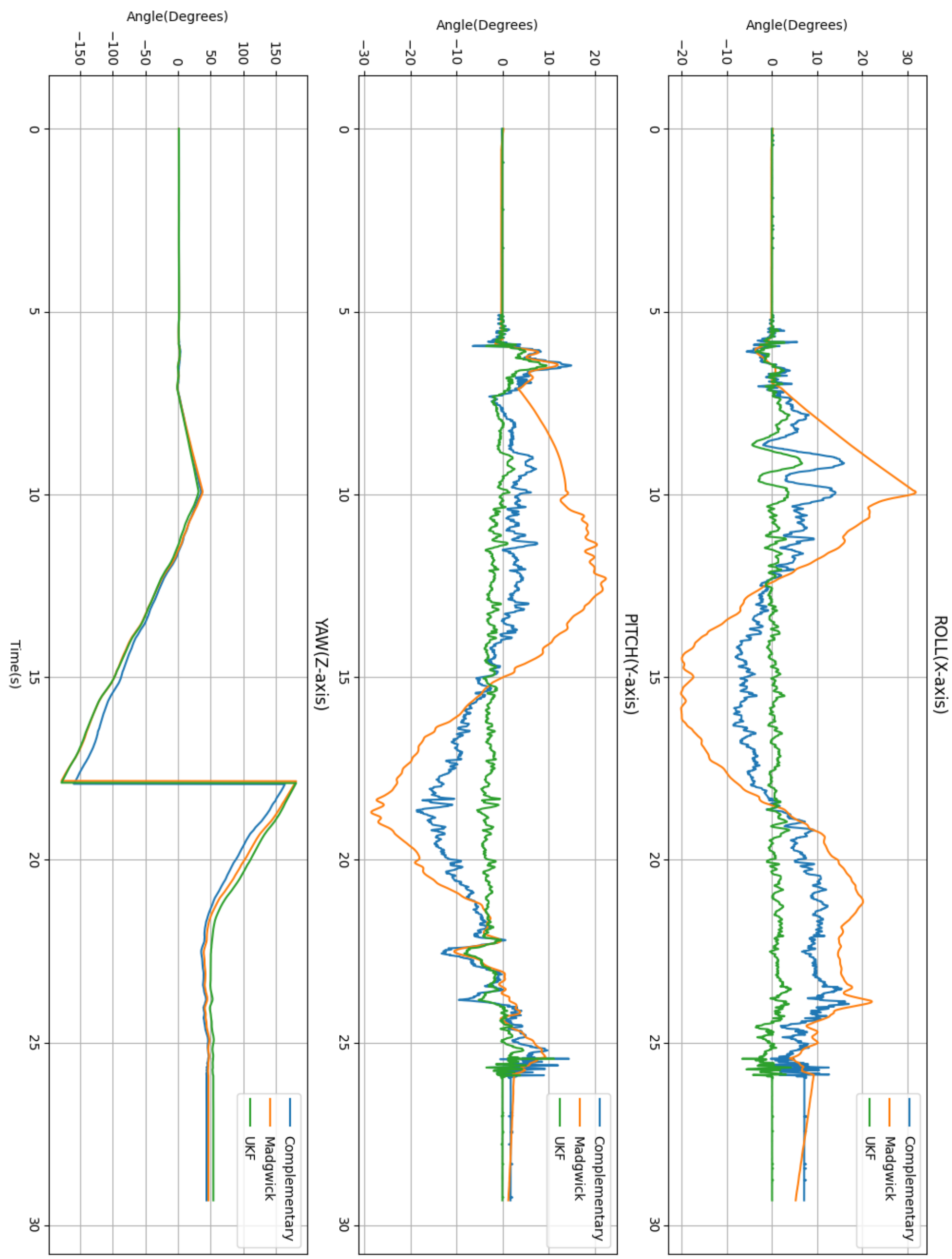

Fig. 9: Comparison of Attitude Estimation for dataset 9

10. TEST DATASET 10

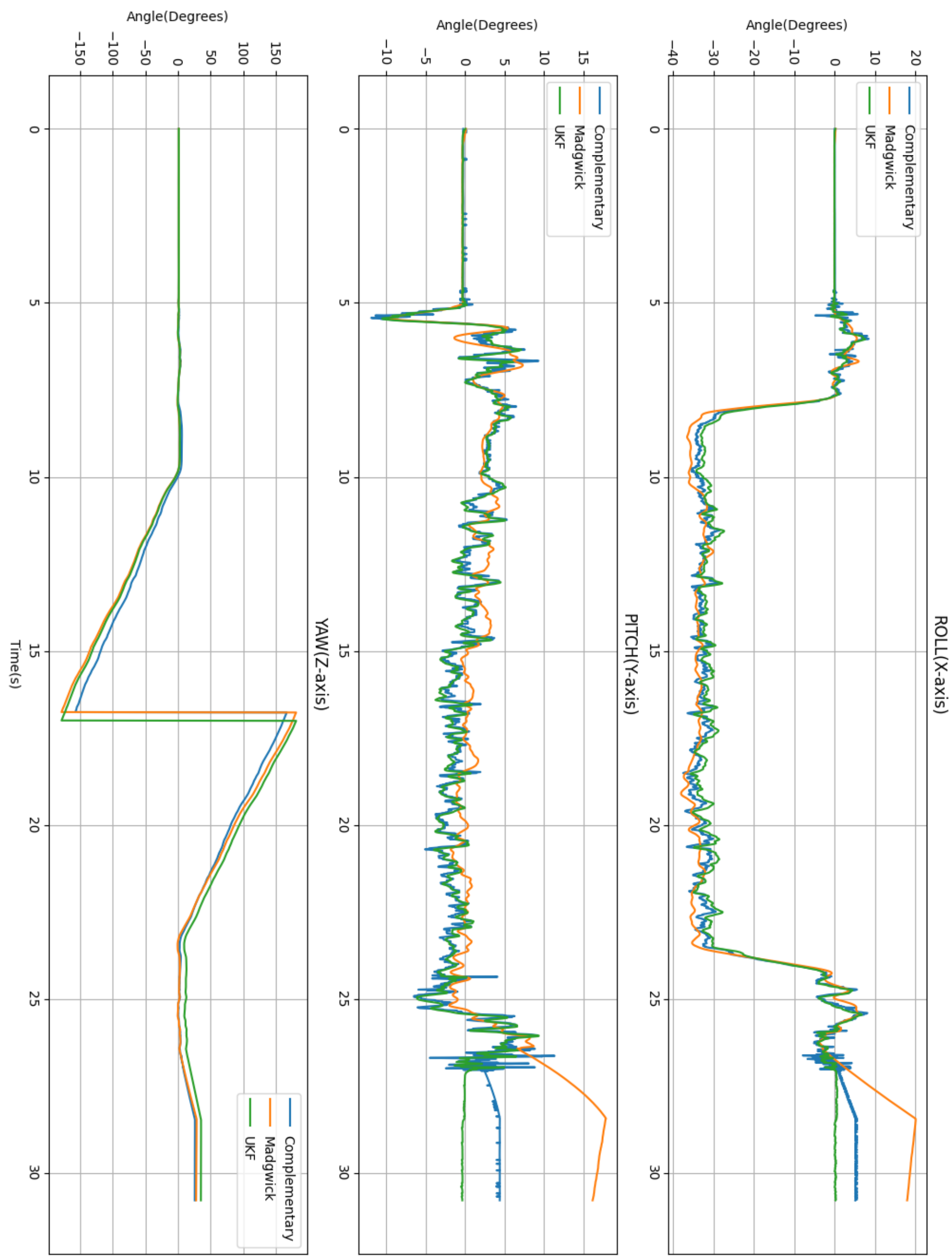

Fig. 10: Comparison of Attitude Estimation for dataset 10