P1 Part A: Madgwick Filter

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I. Problem Statement

The aim of this project was to perform attitude estimation using IMU data. Attitude estimation was done using Madgwick filter.

II. Data conversion and timestamp synchronization

The IMU data consisted of acceleration and angular velocities. The first step in solving the problem statement was to convert the IMU reading to physical dimensions - Acceleration in m/s^2 and angular velocity in rad/s. The acceleration values (a_x, a_y, a_z) were converted to physical units $(\hat{a_x}, \hat{a_y}, \hat{a_z})$mentioned above as follows :

$$\hat{a_x} = \frac{a_x + b_{ax}}{s_x} \tag{1}$$

Finding bias for angular velocities
The gyrometer readings of IMU suffer from noise and bias. i.e. when the IMU is initially at rest, the angular velocities are supposed to be zero. But, the gyrometer still gives non zero values. This is due to the bias in the gyrometer readings. If not compenseted for, the bias can cause a huge drift in the measured values from the actual values of attitude. After a few values, the integration causes the bias to dominate the angular velocity values and will give completely inaccurate values. Thus, we will find gyrometer bias first and compensate for it. For finding bias (b_g), we find average of first few readings (100 values in this case). This value is used in finding physical values of angular velocities as follows:

$$\hat{w} = \frac{3300}{1023} X \frac{\pi}{180} X 0.3 X (w - b_g) \tag{2}$$

Time synchronization between IMU and Vicon
As mentioned in the assignment details, the IMU and Vicon timestamps are not synchronized and thus, we need to do a software synchronization between Vicon and IMU. I have performed a crude synchronization by finding timestamps with minimum difference between IMU and Vicon timestamps. The data values with bigger size are trimmed so that we have same size of data of both Vicon and IMU for further steps

III. Attitude Estimation

As mentioned in the introduction we will try to estimate attitude $[\phi, \theta, \psi]$ using Madgwick filter.
The Madgwick filter makes use of both gyroscope and accelerometer values as follows:
Using Gradient descent on Accelerometer:
The accelerometer values are used for non linear optimization to get estimate of current state. The main idea of using accelerometer or magnetometer data is the assumption that the resultant acceleration on the body od the drone will be dominated be a preexisting field (gravity for accelerometer and Earth's magnetic field for magnetometer). Thus we will use the attitude quternion and acceleration quaternion to align the resultant force with a gravity quaternion.

$$f({}^S_E\hat{q}, {}^E\hat{d}, {}^S\hat{s}) = {}^S_E\hat{q}^* \otimes {}^E\hat{d} \otimes {}^S_E\hat{q} - {}^S\hat{s} \tag{3}$$

Here,
${}^S_E\hat{q}$ - estimated attitude quaternion
${}^E\hat{d}$ - predefined reference field direction (gravity - [0,0,0,1]
${}^S\hat{s}$ - normalised sensor data in sensor frame (measured acceleration values [0, a_x, a_y, a_z])
We use non linear optimization of gradient descent as shown in equation below

$${}^S_E q_{k+1} = {}^S_E q_k - \mu \frac{\nabla f({}^S_E\hat{q}, {}^E\hat{d}, {}^S\hat{s})}{\|f({}^S_E\hat{q}, {}^E\hat{d}, {}^S\hat{s})\|} \tag{4}$$

The gradient of function f is defined as:

$$\nabla f f({}^S_E\hat{q}, {}^E\hat{d}, {}^S\hat{s}) = J^T({}^S_E\hat{q}, {}^E\hat{d}) f({}^S_E\hat{q}, {}^E\hat{d}, {}^S\hat{s}) \tag{5}$$

Here, Jacobian is given as :

$$J = \begin{bmatrix} -2q_3 & 2q_4 & -2q_1 & 2q_2 \\ 2q_2 & 2q_1 & 2q_4 & 2q_3 \\ 0 & -4q_2 & -4q_3 & 0 \end{bmatrix}$$

Function f is defined as :

$$f_g({}^S_E\hat{q}, {}^S\hat{a}) = \begin{bmatrix} 2(q_2q_4 - q_1q_3) - a_x \\ 2(q_2q_1 + q_4q_3) - a_y \\ 2(\frac{1}{2} - q_2^2 - q_3^2) - a_z \end{bmatrix}$$

To get convergance in one step, we need to select correct step size μ as follows:

$$\mu = \alpha \|{}^S_E\dot{q}_{\omega,t}\| \Delta t, \alpha > 1$$

For above set of equations for attitude estimation, we need an initial condition for the numerical integration. For "Train" testcase, we have Vicon data available. Thus, we use the Vicon data at time, t = 0. For "Test" datasets, we use initial angles for all 3 Euler angles to be 0 degrees. For further timestamps, k+1, we use previous attitude estimate at timestamp k using Madgwick filter.
Orientation increment using gyroscope
We use gyroscope data to find the change in orientation for time step Δt as follows:

$${}^S_E\dot{q}_{\omega,t+1} = \frac{1}{2}{}^I_W \hat{q}_{est,t} \otimes [0, {}^I\omega_{t+1}] \tag{6}$$

Fuse the measurements
Both the gyroscope attitude increment and accelerometer opti-

mize estimates are fused together to estimate attitude at current time step as follows:

$$ {}^I_W\dot{q}_{est,t+1} = {}^I_W \dot{q}_{\omega,t+1} + {}^I_W q_{\nabla,t+1} \tag{7}$$

$$ {}^I_W q_{est,t+1} = {}^I_W \hat{q}_{est,t} + {}^I_W \dot{q}_{est,t+1}\Delta t \tag{8}$$

Gradient coefficient β:
The only parameter which can be manually tuned in the Madgwick filter is the coefficient of multiplication of gradient in accelerometer gradient descent. β defined the trust factor between accelerometer and gyroscope. During our attitude estimation, we found the accelerometer values were more reliable and thus we set $\beta = 0.8$. But in some case, due to computation inaccuracies in implementation of Quaternion class and its member and non member functions, we were getting some drift in accelerometer attitude estimation, thus we also checked attitude estimation values with $\beta = 0.4$. We are including the results for both the beta values in the graphs in Results sections below:

IV. Video Link for RotPlot

Google Drive with RotPlot videos

V. Results

β = 0.8 Train datasets

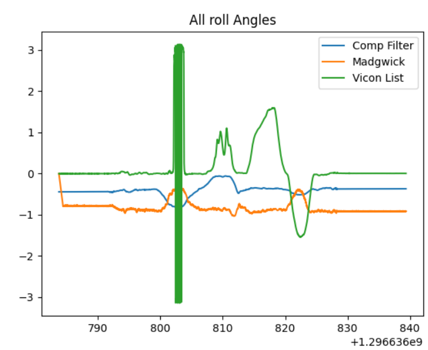

Fig. 1: Madgwick Filter Roll Angle Estimation

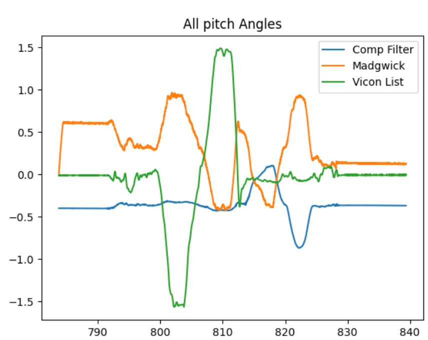

Fig. 2: Madgwick Filter Pitch Angle Estimation

Similarly the attitude estimation using all 3 methods is shown in plots below for other datasets:

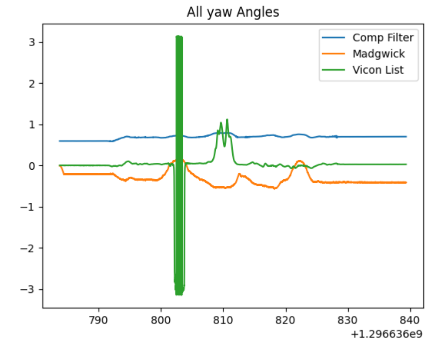

Fig. 3: Madgwick Filter Yaw Angle Estimation

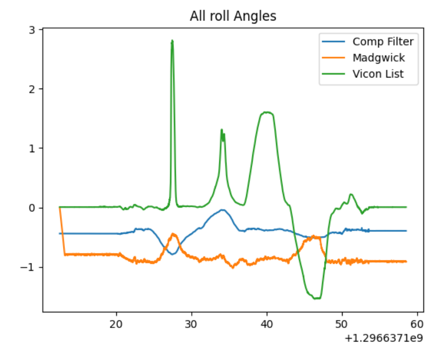

Fig. 4: Dataset 2 Madgwick Filter Roll Angle Estimation

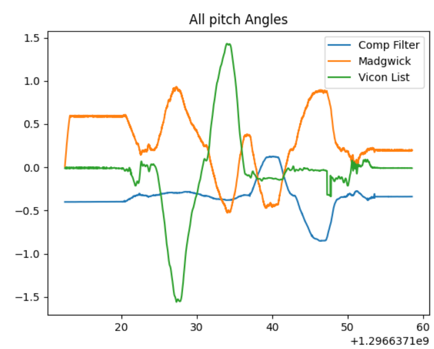

Fig. 5: Dataset 2 Madgwick Filter Pitch Angle Estimation

Fig. 6: Dataset 2 Madgwick Filter Yaw Angle Estimation

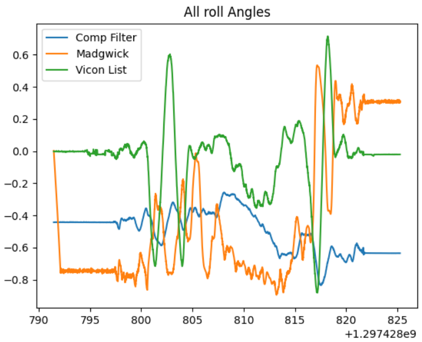

Fig. 7: Dataset 3 Madgwick Filter Roll Angle Estimation

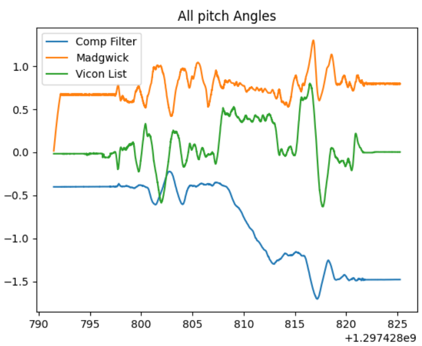

Fig. 8: Dataset 3 Madgwick Filter Pitch Angle Estimation

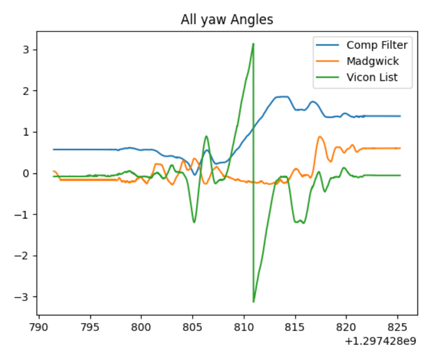

Fig. 9: Dataset 3 Madgwick Filter Yaw Angle Estimation

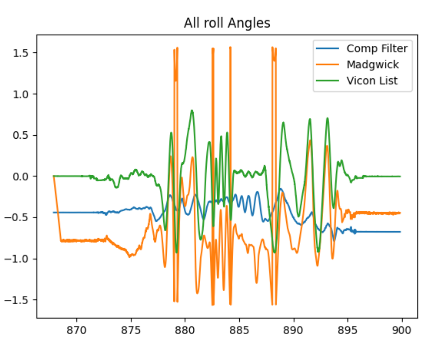

Fig. 10: Dataset 5 Madgwick Filter Roll Angle Estimation

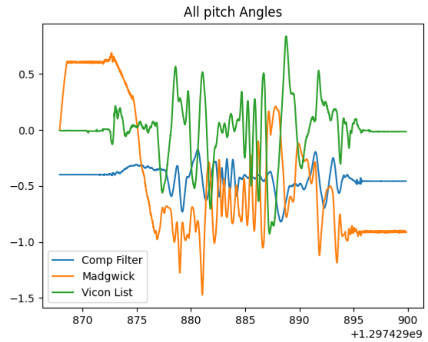

Fig. 11: Dataset 5 Madgwick Filter Pitch Angle Estimation

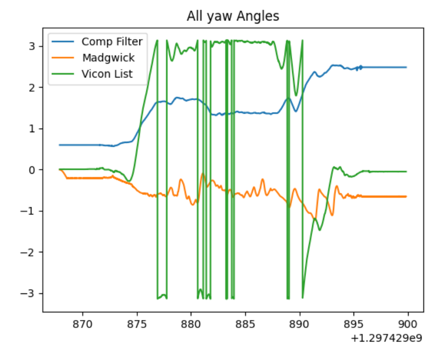

Fig. 12: Dataset 5 Madgwick Filter Yaw Angle Estimation

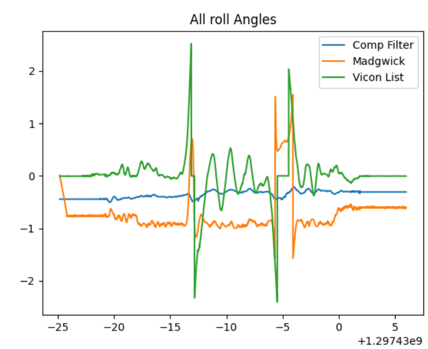

Fig. 13: Dataset 6 Madgwick Filter Roll Angle Estimation

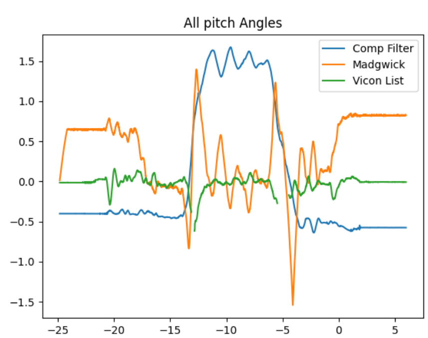

Fig. 14: Dataset 6 Madgwick Filter Pitch Angle Estimation

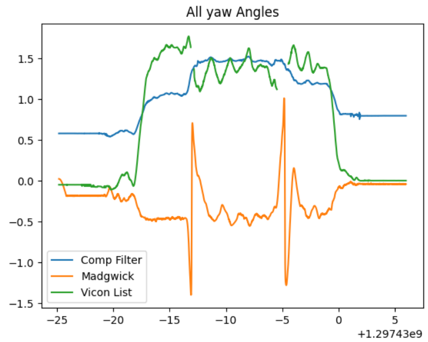

Fig. 15: Dataset 6 Madgwick Filter Yaw Angle Estimation

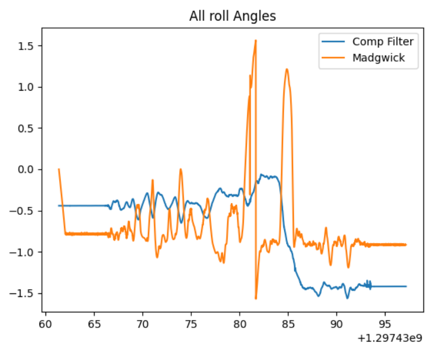

Fig. 16: Dataset 7 Madgwick Filter Roll Angle Estimation

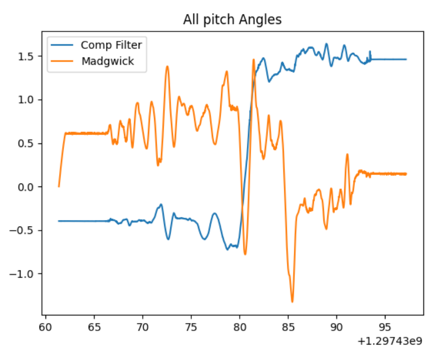

Fig. 17: Dataset 7 Madgwick Filter Pitch Angle Estimation

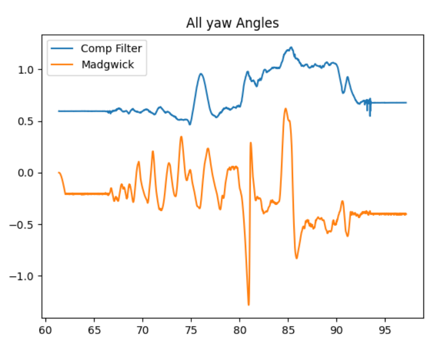

Fig. 18: Dataset 7 Madgwick Filter Yaw Angle Estimation

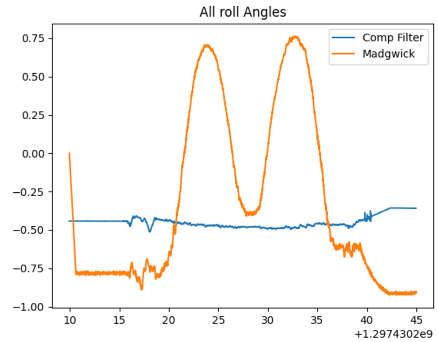

Fig. 19: Dataset 8 Madgwick Filter Roll Angle Estimation

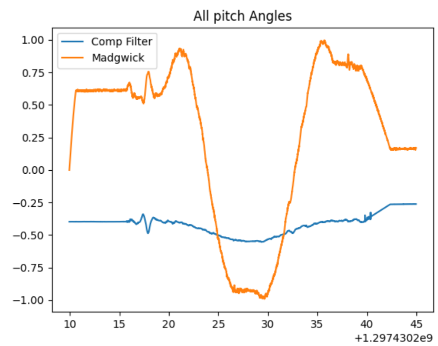

Fig. 20: Dataset 8 Madgwick Filter Pitch Angle Estimation

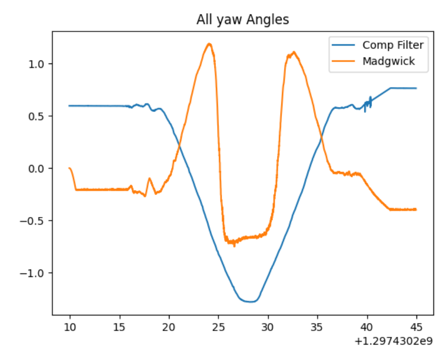

Fig. 21: Dataset 8 Madgwick Filter Yaw Angle Estimation

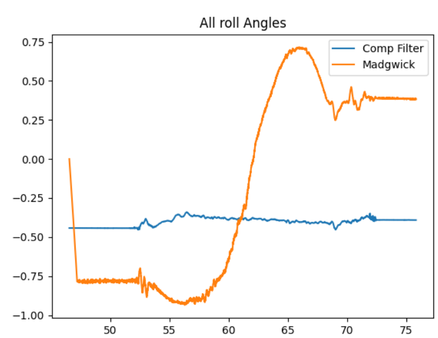

Fig. 22: Dataset 9 Madgwick Filter Roll Angle Estimation

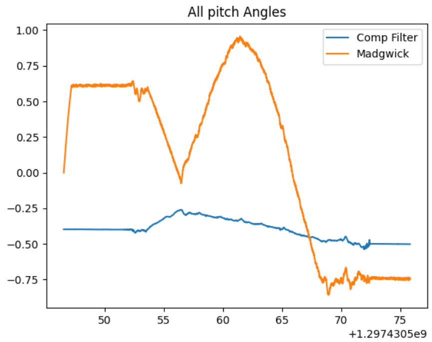

Fig. 23: Dataset 9 Madgwick Filter Pitch Angle Estimation

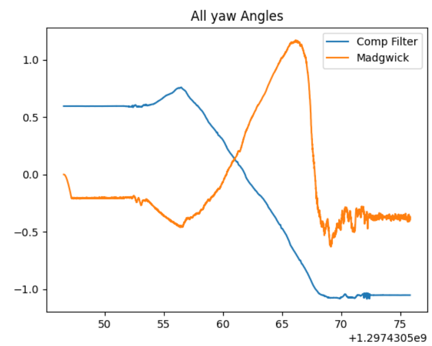

Fig. 24: Dataset 9 Madgwick Filter Yaw Angle Estimation

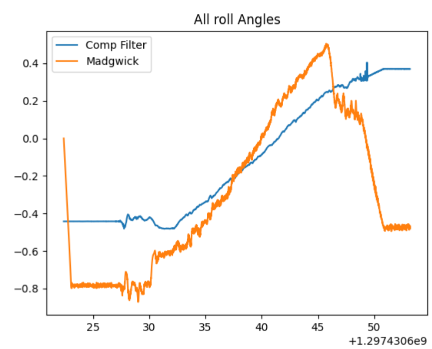

Fig. 25: Dataset 10 Madgwick Filter Roll Angle Estimation

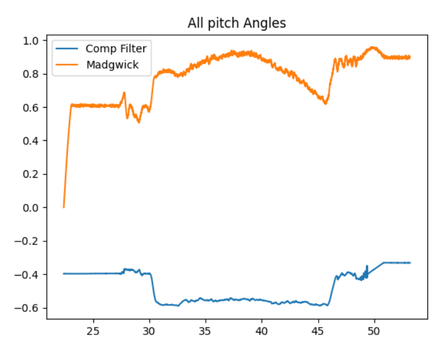

Fig. 26: Dataset 10 Madgwick Filter Pitch Angle Estimation

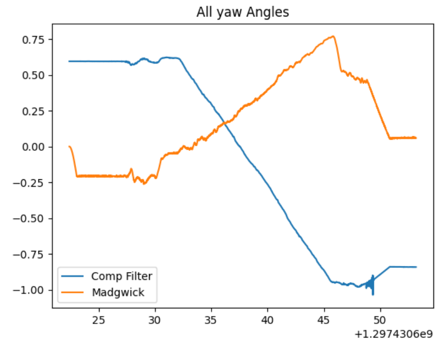

Fig. 27: Dataset 10 Madgwick Filter Yaw Angle Estimation

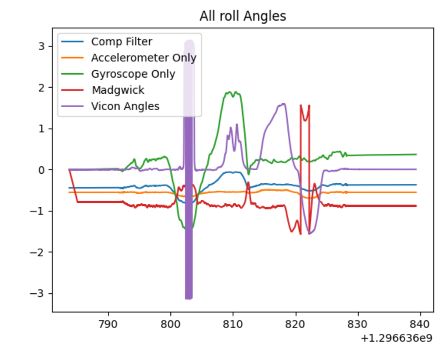

Fig. 28: Madgwick Filter Roll Angle Estimation

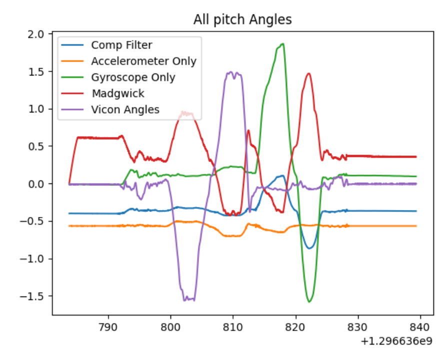

Fig. 29: Madgwick Filter Pitch Angle Estimation

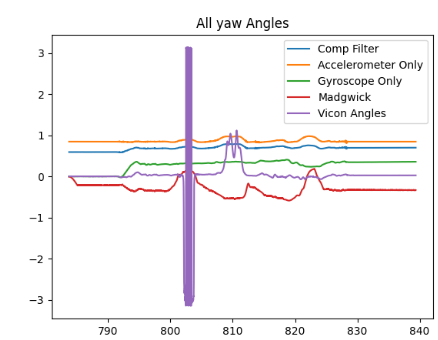

Fig. 30: Madgwick Filter Yaw Angle Estimation

β **= 0.8 Test datasets** - fig 16 - fig 27

β **= 0.4 Train datasets** - fig 18 - fig 42

Similarly the attitude estimation using all 3 methods is shown in plots below for other datasets:

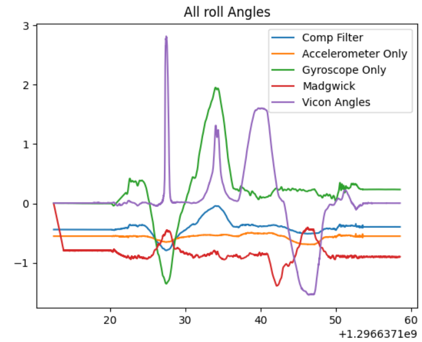

Fig. 31: Dataset 2 Madgwick Filter Roll Angle Estimation

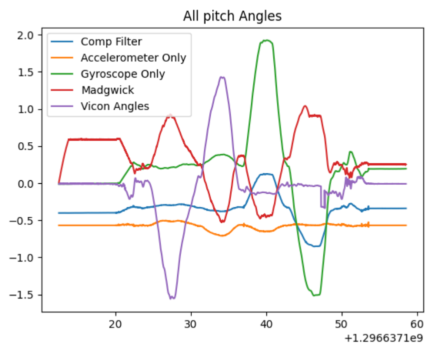

Fig. 32: Dataset 2 Madgwick Filter Pitch Angle Estimation

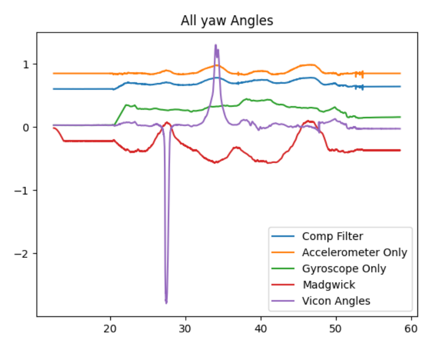

Fig. 33: Dataset 2 Madgwick Filter Yaw Angle Estimation

β **= 0.8 Test datasets**

VI. References

1. An efficient orientation filter for inertial and inertial/magnetic sensor arrays Sebastian O.H. Madgwick April 30, 2010

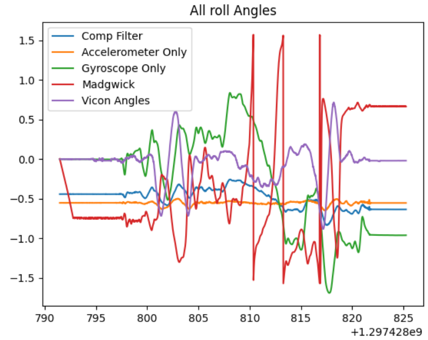

Fig. 34: Dataset 3 Madgwick Filter Roll Angle Estimation

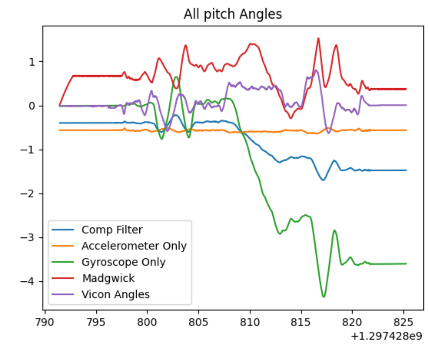

Fig. 35: Dataset 3 Madgwick Filter Pitch Angle Estimation

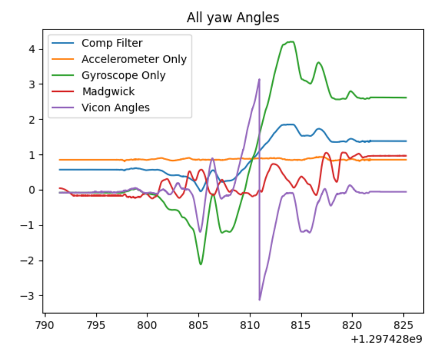

Fig. 36: Dataset 3 Madgwick Filter Yaw Angle Estimation

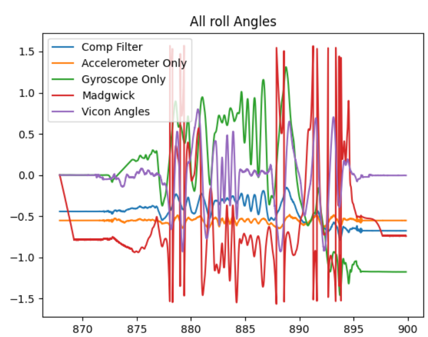

Fig. 37: Dataset 5 Madgwick Filter Roll Angle Estimation

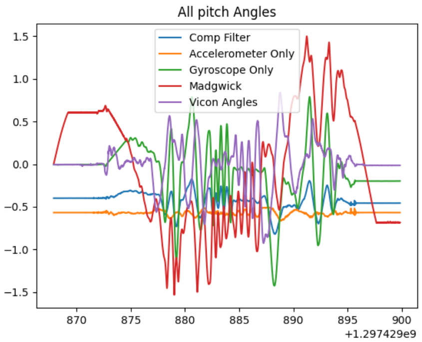

Fig. 38: Dataset 5 Madgwick Filter Pitch Angle Estimation

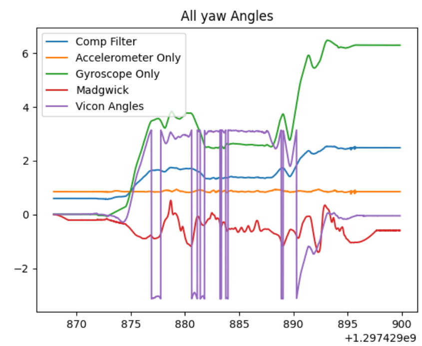

Fig. 39: Dataset 5 Madgwick Filter Yaw Angle Estimation

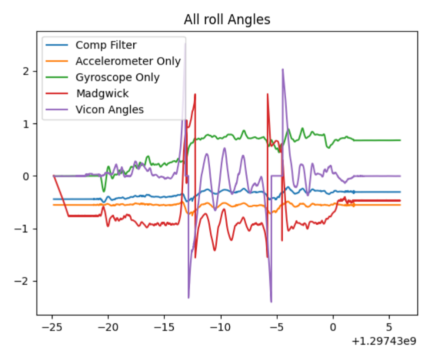

Fig. 40: Dataset 6 Madgwick Filter Roll Angle Estimation

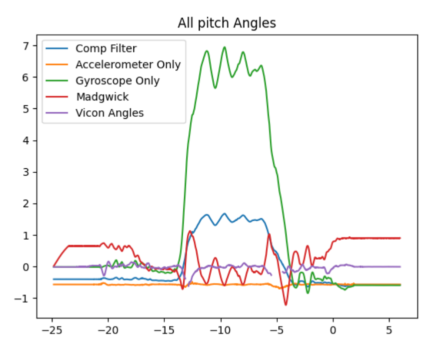

Fig. 41: Dataset 6 Madgwick Filter Pitch Angle Estimation

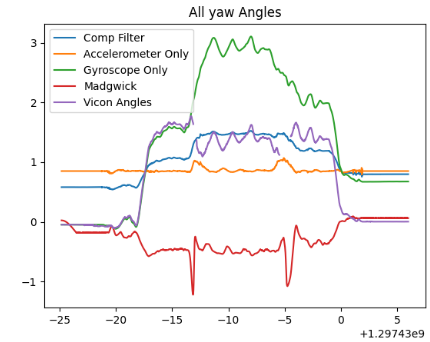

Fig. 42: Dataset 6 Madgwick Filter Yaw Angle Estimation

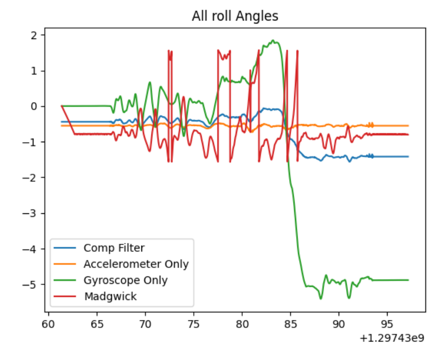

Fig. 43: Dataset 7 Madgwick Filter Roll Angle Estimation

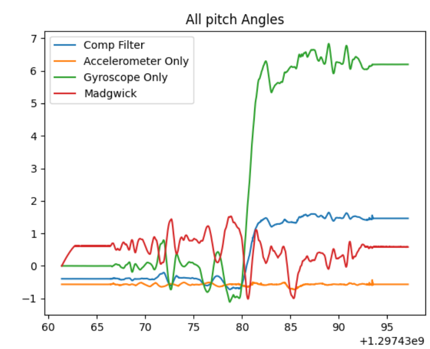

Fig. 44: Dataset 7 Madgwick Filter Pitch Angle Estimation

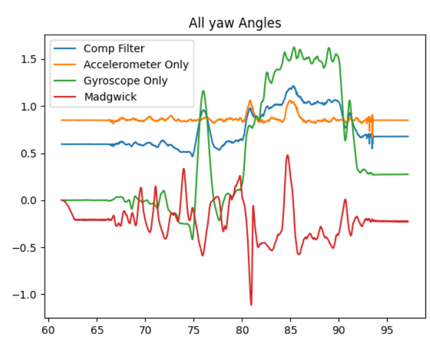

Fig. 45: Dataset 7 Madgwick Filter Yaw Angle Estimation

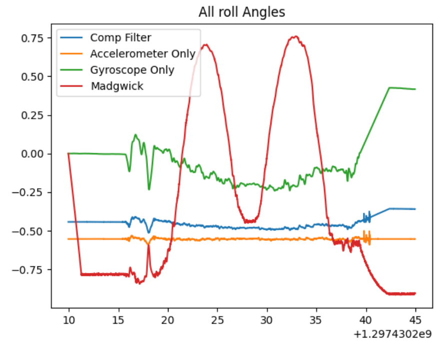

Fig. 46: Dataset 8 Madgwick Filter Roll Angle Estimation

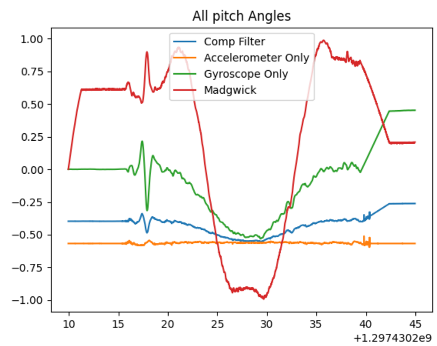

Fig. 47: Dataset 8 Madgwick Filter Pitch Angle Estimation

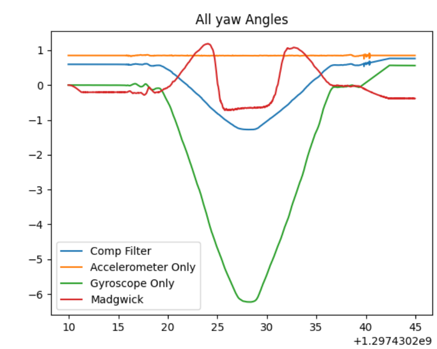

Fig. 48: Dataset 8 Madgwick Filter Yaw Angle Estimation

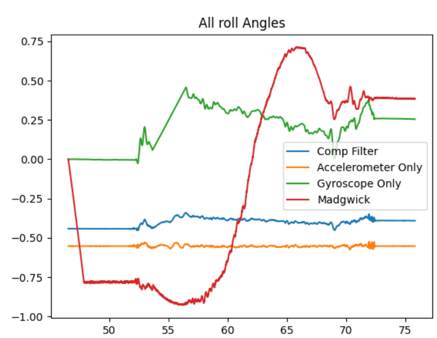

Fig. 49: Dataset 9 Madgwick Filter Roll Angle Estimation

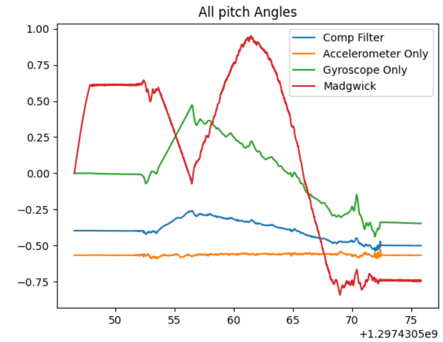

Fig. 50: Dataset 9 Madgwick Filter Pitch Angle Estimation

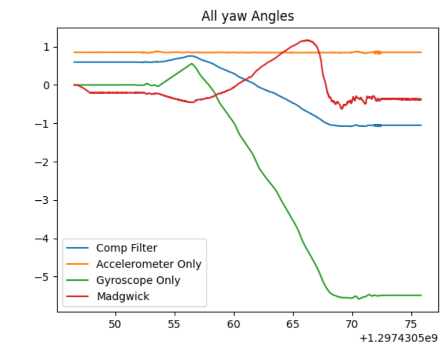

Fig. 51: Dataset 9 Madgwick Filter Yaw Angle Estimation

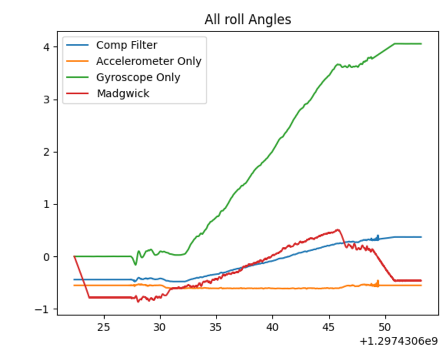

Fig. 52: Dataset 10 Madgwick Filter Roll Angle Estimation

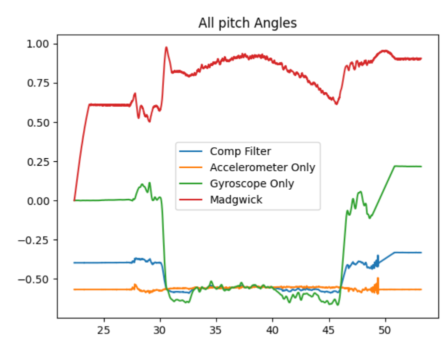

Fig. 53: Dataset 10 Madgwick Filter Pitch Angle Estimation

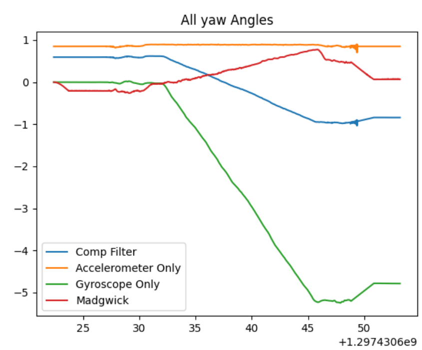

Fig. 54: Dataset 10 Madgwick Filter Yaw Angle Estimation