Attitude Estimation using Madgwick Filter on 6-DoF IMU

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***Abstract*—This project presents the implementation of Madgwick filter to estimate the 3D orientation of the IMU (ArduIMU+ V2) by reading the acceleration and the gyroscope values given by it. A complementary filter is also implemented to fuse the values from the accelerometer and the gyroscope to obtain the orientation. The orientation obtained by the filters is then compared with the ground truth from the vicon motion capture system.**

$$R = R_z(\alpha)\,R_y(\beta)\,R_x(\gamma) = \overset{\text{yaw}}{\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}}\overset{\text{pitch}}{\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}}\overset{\text{roll}}{\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\gamma & -\sin\gamma\\ 0 & \sin\gamma & \cos\gamma\end{bmatrix}}$$
$$= \begin{bmatrix}\cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma-\sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma+\sin\alpha\sin\gamma\\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma+\cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma-\cos\alpha\sin\gamma\\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma\end{bmatrix}$$

Fig. 1. 3D Rotation

I. Implementation

A. Reading and Appropriating the Data

The data is obtained from six degree IMU, 3-axis gyroscope and 3-axis accelerometer fitted on a drone. The orientation of the IMU recorded from the VICON motion capture is also provided. The data for each correspondence is provided in a .mat file. The IMU data exists as:

$$[a_x, a_y, a_z, w_z, w_x, w_y]$$

The VICON data meanwhile stores the timestamps **ts** and the orientation in a 3x3 Rotation matrix denoting **Z-Y-X** Euler angles for N time instances. The parameter values for the IMU are also provided in a 2x3 vector containing the Scale and Bias values.

The acceleration has been converted into m/s^2. The formula implemented in the code is:

$$\bar{a} = (a \cdot scale + bias) \cdot g$$

The angular rates have been converted into rad/s^{-1}.

$$\bar{\omega} = \frac{3300}{1023} \cdot \frac{\pi}{180} \cdot (\omega - b_g)$$

The bias for angular rate conversion is obtained by calculating the average of the first couple hundred values of each angular rate from the IMU data.

B. Implementation of the Methods to estimate Orientations

Methodology

We use four methods to estimate the orientation of the IMU. The first method uses only the gyroscope readings for attitude estimation, while the second uses the accelerometer readings. We also implement a Complementary filter and Madgwick filter by fusing the readings from both gyroscope and accelerometer.

Rotation Matrix using R-P-Y Euler Angles Method

The attitude is calculated from the roll, pitch and yaw rate by using the 3D rotation matrix formula notation in Fig 1.

1) **Orientation From Gyroscope Measurements:**

   Using Integration:

   - The function iterates through the IMU timestamps and gyroscope measurements obtained from the IMU data.
   - For each timestamp, the time difference between the current and previous timestamps is calculated.
   - The orientations are updated using the roll, pitch, and yaw angles calculated above w_x, w_y, w_z.

2) **Orientation from Accelerometer Measurements**:

   - The second method used only the accelerometer readings for attitude estimation.
   - The orientation is returned as well as the roll, pitch and yaw values stored as a vector are returned.
   - The formula is used keeping in mind IMU is only rotating and that the acceleration due to gravity is in the Z-axis.

$$\text{Roll }, \phi = \tan^{-1}\left(\frac{a_y}{\sqrt{a_x^2 + a_z^2}}\right)$$

$$\text{Pitch }, \theta = \tan^{-1}\left(\frac{-a_x}{\sqrt{a_y^2 + a_z^2}}\right)$$

$$\text{Yaw }, \psi = \tan^{-1}\left(\frac{\sqrt{a_x^2 + a_y^2}}{a_z}\right)$$

3) **Complementary Filter**:

- We low pass the accelerometer measurement data as described below. The alpha for Low Pass filter is taken as 0.2

$$\hat{\mathbf{a}}_{t+1} = (1-\alpha)\mathbf{a}_{t+1} + \alpha\hat{\mathbf{a}}_t$$

Fig. 2. Accelerometer LPF

- We high pass the gyroscope measurement data as described below. The alphs for High Pass filter is taken as 0.2.

$$\hat{\omega}_{t+1} = (1-\alpha)\hat{\omega}_t + (1-\alpha)(\omega_{t+1} - \omega_t)$$

Fig. 3. Gyroscope HPF

- The two measurements are fused into one using a α value that weighs the two. The alpha for complementary filter is taken as 0.5.
- The output orientation is stored for plotting.

4) **Madgwick Filter**:

The newer method used for the above estimated attitudes is **Madgwick** filter. This filter formulates the problem in quaternion space.

- We begin by calculating the readings for orientation obtained only from the gyroscope and only from the accelerometer, in the quaternion notation.
- Normalise the quaternion notation orientation we calculated in the above steps using formula in Fig 2.
- The second step is to perform the orientation increments from the accelerometer. Let our initial estimate begin at the quaternion [1 0 0 0].

$$\begin{aligned}
q_w &= \cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) + \sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\
q_x &= \sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) - \cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\
q_y &= \cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right) + \sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\
q_z &= \cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right) - \sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right)
\end{aligned}$$

Fig. 4. Euler Angles to Quaternion Conversion

- We calculate the orientation estimate from the gyroscope using the formula below.

$${}^I_W\dot{\mathbf{q}}_{\omega,t+1} = \frac{1}{2} W\hat{\mathbf{q}}_{\text{est},t} \otimes \begin{bmatrix} 0 \\ {}^I\omega_{t+1} \end{bmatrix} \tag{1}$$

- The function **f** formulated for the compliance of the values and the Jacobian (**J**) are calculated and their product is used to get our gradient from our previous estimate.

$$\begin{aligned}
&\nabla f\left({}^I_W\hat{\mathbf{q}}_{\text{est},t},\ {}^W\hat{\mathbf{g}},\ {}^I\hat{\mathbf{a}}_{t+1}\right)\\
&= J^T\left({}^I_W\hat{\mathbf{q}}_{\text{est},t},\ {}^W\hat{\mathbf{g}}\right)\nabla f\left({}^I_W\hat{\mathbf{q}}_{\text{est},t},\ {}^W\hat{\mathbf{g}},\ {}^I\hat{\mathbf{a}}_{t+1}\right)
\end{aligned} \tag{2}$$

where

$$J\left({}^I_W\hat{\mathbf{q}}_{\text{est},t}, {}^W\hat{\mathbf{g}}\right) = \begin{bmatrix} -2q_3 & 2q_4 & -2q_1 & 2q_2 \\ 2q_2 & 2q_1 & 2q_4 & 2q_3 \\ 0 & -4q_2 & -4q_3 & 0 \end{bmatrix}$$

and

$$f\left({}^I_W\hat{\mathbf{q}}_{\text{est},t}, {}^W\hat{\mathbf{g}}, {}^I\hat{\mathbf{a}}_{t+1}\right) = \begin{bmatrix} 2\left(q_2q_4 - q_1q_3\right) - a_x \\ 2\left(q_1q_2 + q_3q_4\right) - a_y \\ 2\left(\frac{1}{2} - q_2^2 - q_3^2\right) - a_z \end{bmatrix} \tag{3}$$

- **Update Step:** The normalised gradient is multiplied by β and that product is subtracted from the new gyroscope measurement estimate.

The term **beta** is the trust ratio between the accelerometer and gyroscope values and is set to 0.09.(Manually tuned)

- Calculate use the product of the update step and the difference in the time step.
- The new estimate is given as the sum of the normalised previous estimate and the product above. See the formula below.

$$ {}^{I}_{W}\dot{\mathbf{q}}_{\text{est},t+1} = {}^{I}_{W}\ \dot{\mathbf{q}}_{\omega,t+1} + {}^{I}_{W}\ \mathbf{q}_{\nabla,t+1} $$

- Repeat the steps at each time instant.
- Estimates calculated at each time instant are converted to the Euler angle representation.

$$ \sin(pitch) = 2 \cdot (w \cdot y - z \cdot x) $$

$$ pitch = \begin{cases} \frac{\pi}{2} & \text{if } \sin(pitch) > 0 \\ -\frac{\pi}{2} & \text{if } \sin(pitch) < 0 \\ \arcsin(\sin(pitch)) & \text{otherwise} \end{cases} $$

$$ \sin(yaw + pitch) = 2 \cdot (w \cdot z + x \cdot y) $$

$$ \cos(yaw + pitch) = 1 - 2 \cdot (y^2 + z^2) $$

$$ yaw = \arctan 2(\sin(yaw + pitch), \cos(y $$

$$ \sin(roll) = 2 \cdot (w \cdot x - y \cdot z) $$

$$ roll = \begin{cases} \frac{\pi}{2} & \text{if } \sin(roll) > 0 \\ -\frac{\pi}{2} & \text{if } \sin(roll) < 0 \\ \arcsin(\sin(roll)) & \text{otherwise} \end{cases} $$

II. Results

The beta and alpha values for the filters are tuned such that it matches closely the VICON data. Please see the figures attached here. The link to the videos obtained after running the `rotplot.py` and can be referred to with the name of the data set where 1-6 is for the given the data set and 7-10 is the test sets.

LINK: https://drive.google.com/drive/folders/1diiW5OvQsa-m4cf5huYon-TeDV9fEZU3

III. References

1 https://nitinjsanket.github.io/tutorials/attitudeest/

2 ENAE788M: YouTube

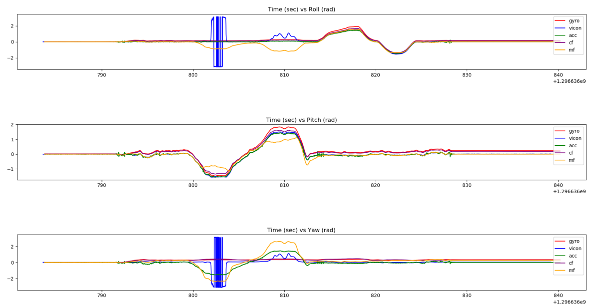

Fig. 5. Plots for Dataset 1

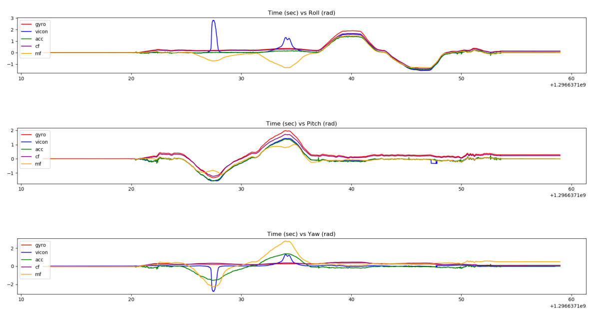

Fig. 6. Plots for Dataset 2

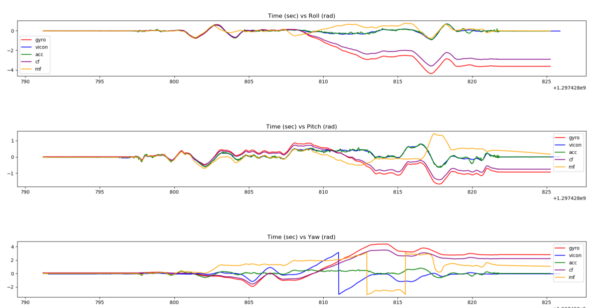

Fig. 7. Plots for Dataset 3

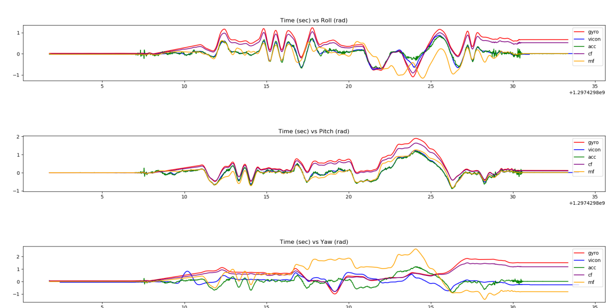

Fig. 8. Plots for Dataset 4

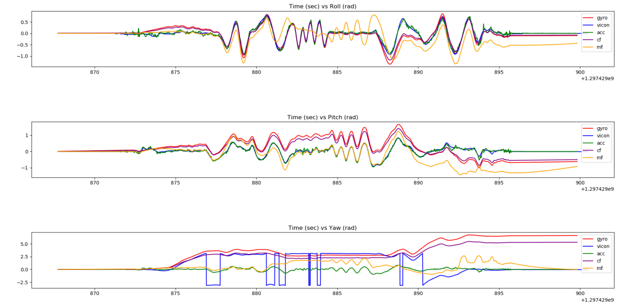

Fig. 9. Plots for Dataset 5

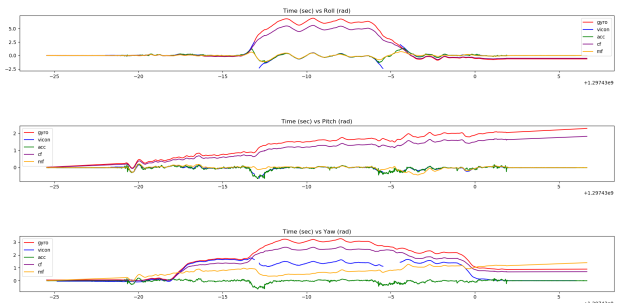

Fig. 10. Plots for Dataset 6

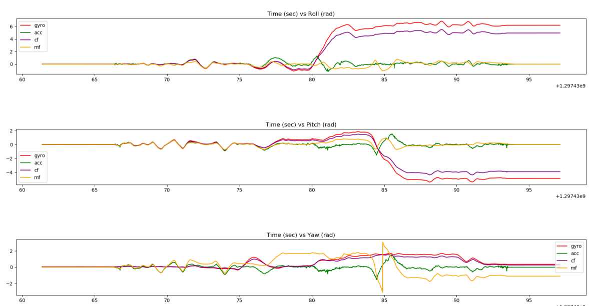

Fig. 11. Plots for Test Dataset 1

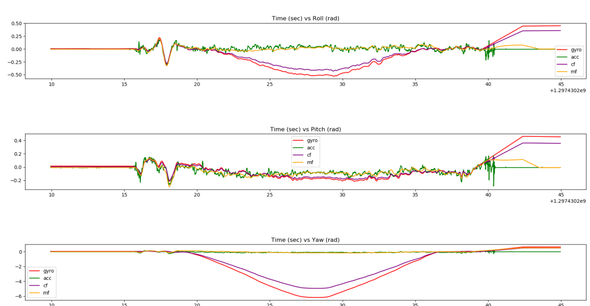

Fig. 12. Plots for Test Dataset 2

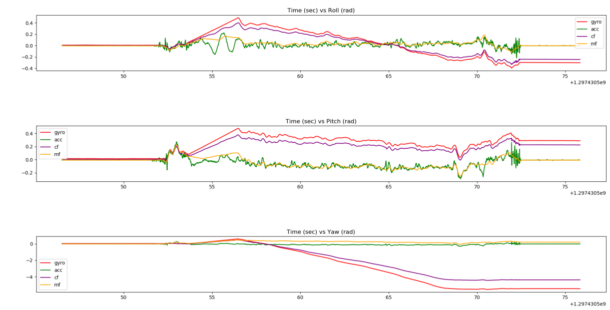

Fig. 13. Plots for Test Dataset 3

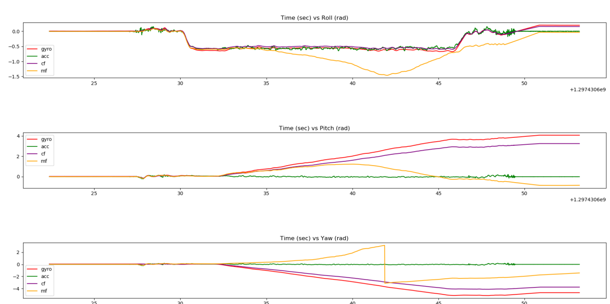

Fig. 14. Plots for Test Dataset 4