Homework 1 - AutoCalib

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***Abstract*—Camera calibration involves estimating the parameters of a lens and image sensor of an image camera that includes camera intrinsic, extrinsic, and distortion coefficients. These parameters are essential for correcting lens distortion, measuring the size of objects in the world, and localizing the camera.**

I. Introduction

To estimate the camera parameters, we need to have 3D world points and corresponding 2D image points. We can obtain these correspondences using multiple images of a checkerboard (a calibration pattern). Using these correspondences, we can solve for the camera parameters such as the intrinsic, extrinsic, and distortion coefficients. For this report, we have been provided with 13 images of a checkerboard captured using a Pixel XL phone. The grid size of the checkerboard is 11×8 with each square of length 21.1mm. Zhang's method [1] is used to calibrate the camera, which will be described in the next section.

II. Methodology

A. Initial Estimation of Intrinsic Parameters

Intrinsic parameters of the camera include principal points (u_0, v_0), scaling factors (α, β), and skewness (γ). The homography matrix will capture the transformation between the world points and the corresponding 2D image points (checkerboard square corners). *cv2.findChessboardCorners* was used to find the corner coordinates in an image excluding the squares at the border. Therefore, there will be 54 corners for each image (9×6). The world points are calculated using the number of squares along the width and the length as well as the width of each square. Depth (Z) is assumed to be 0. After estimating the world corners, the homography matrix (H) for each image is calculated using the corresponding 2D image points. The relation between the 2D image points ($c = \begin{bmatrix} u & v & 1 \end{bmatrix}^T$) and the world points ($C = \begin{bmatrix} X & Y & Z & 1 \end{bmatrix}^T$) is given by:

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} R|t \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{1}$$

where s is the scale, K is the calibration matrix, $\begin{bmatrix} R|t \end{bmatrix}$ are the extrinsics. Since $Z = 0$ and the checkerboard is a plane, we can further simplify Equation 1.

$$\begin{aligned} s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} &= K \begin{bmatrix} R|t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} && (2) \\ &= K \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} && (3) \\ &= K \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} && (4) \\ &= KH \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} && (5) \end{aligned}$$

where H is the homography matrix. Let $H = \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}$. We can have a closed solution using Cholesky Decomposition for K using equation 7

$$h_1^T (K^{-T} K^{-1}) h_2 = 0 \tag{6}$$
$$h_1^T (K^{-T} K^{-1}) h_1 = h_2^T (K^{-T} K^{-1}) h_2 \tag{7}$$

Let $B = K^{-T} K^{-1}$ be a symmetric matrix $\in R^{3 \times 3}$. We can find the solution for B using the following system of equations:

$$h_i^T B h_j = v_{ij}^T b \tag{8}$$
$$\begin{bmatrix} v_{12}^T \\ (v_{11} - v_{22})^T \end{bmatrix} b = 0 \tag{9}$$
$$v_{ij} = \begin{bmatrix} h_{i1} h_{j1} \\ h_{i1} h_{j2} + h_{i2} h_{j1} \\ h_{i2} h_{j2} \\ h_{i3} h_{j1} + h_{i1} h_{j3} \\ h_{i3} h_{j2} + h_{i2} h_{j3} \\ h_{i3} h_{j3} \end{bmatrix} \tag{10}$$

where $b = \begin{bmatrix} B_{11} & B_{12} & B_{22} & B_{13} & B_{23} & B_{33} \end{bmatrix}^T$ Solving these equations using singular value decomposition, we can get the b matrix and thus the intrinsic matrix K. More specifically, we can calculate the focal length, scaling factors, and skewness as follows:

$$v_0 = \frac{B_{12}B_{13} - B_{11}B_{23}}{B_{11}B_{22} - B_{12}^2} \tag{11}$$

$$\lambda = B_{33} - \frac{B_{13}^2 + v_0(B_{12}B_{13} - B_{11}B_{23})}{B_{11}} \tag{12}$$

$$\alpha = \sqrt{\lambda / B_{11}} \tag{13}$$

$$\beta = \sqrt{\frac{\lambda B_{11}}{B_{11}B_{22} - B_{12}^2}} \tag{14}$$

$$\gamma = \frac{-B_{12}\alpha^2\beta}{\lambda} \tag{15}$$

$$u_0 = \frac{\gamma v_0}{\beta} - \frac{B_{13}\alpha^2}{\lambda} \tag{16}$$

Using these equations, the calibration matrix K can be constructed as:

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$\tag{18}$$

B. Extrinsic Parameters

Now that we have an initial estimate of the intrinsic parameters, we can recover the extrinsic parameters using the following:

$$r_1 = \psi K^{-1} h_1 \tag{19}$$

$$r_2 = \psi K^{-1} h_2 \tag{20}$$

$$r_3 = r_1 \times r_2 \tag{21}$$

$$r_1 = \psi K^{-1} h_3 \tag{22}$$

$$\psi = \frac{1}{||K^{-1} h_1||} \tag{23}$$

C. Distortion and Reprojection error

Using the extrinsic parameters, we can project back the corners onto the image plane and calculate the projection error between the projected corners and the detected corners. This will be used to calculate the optimal intrinsic parameters as well as the distortion coefficients which were assumed to be zero. The error function that is to be minimized is given by:

$$E = \Sigma_{i=1}^{n} \Sigma_{j=1}^{m} ||c_{ij} - c^{`}(K, R_i, t_i, C_j)||_2 \tag{24}$$

If (u, v) are the distortion-free image coordinates, then the corresponding distorted image coordinates (u', v') are given by:

$$u' = u + (u - u_0)(k_1(x^2 + y^2) + k_2(x^2 + y^2)^2 \tag{25}$$

$$v' = v + (v - v_0)(k_1(x^2 + y^2) + k_2(x^2 + y^2)^2 \tag{26}$$

$$\tag{27}$$

where (x, y) are the corresponding projected corners. *scipy.optimize.least_squares* was used to minimize the reprojected error E.

III. Results

A. Calibration Matrix, Distortion Coefficients and Reprojection Error

The calibration matrix before and after optimization was found to be:

$$K = \begin{bmatrix} 2065.25652 & -2.93974703 & 764.676160 \\ 0.0 & 2053.48352 & 1362.76925 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{28}$$

$$K_{opt} = \begin{bmatrix} 2065.25636 & -2.93989308 & 764.671419 \\ 0.0 & 2053.48303 & 1362.76360 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{29}$$

The mean reprojection error and the total reprojection error have been reported in Table I.

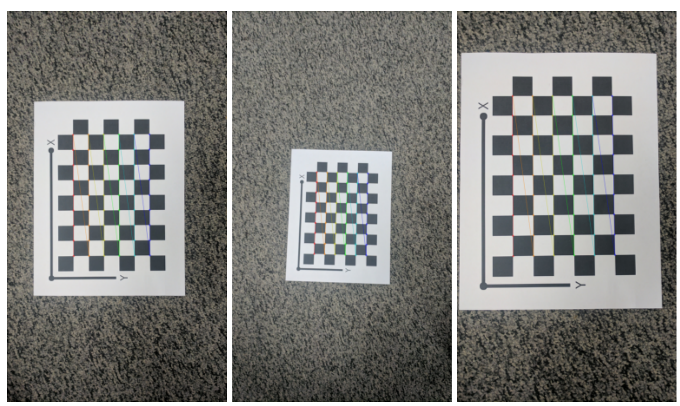
	Before Optimization	After Optimization
Total	569.3586	566.4033
Mean	0.751869	0.7473123

TABLE I: Reprojection Error

The distortion coefficients, k, were found to be: $\begin{bmatrix} 2.04482639e-03 & -1.06395874e-02 \end{bmatrix}$

B. Detected and Reprojected Corners

For visualizing undistorted images, *cv2.undistort* was used with optimized camera intrinsics and distortion parameters. Initial corners detected are illustrated in Figure 1 while the corresponding undistorted images are illustrated in Figure 2.

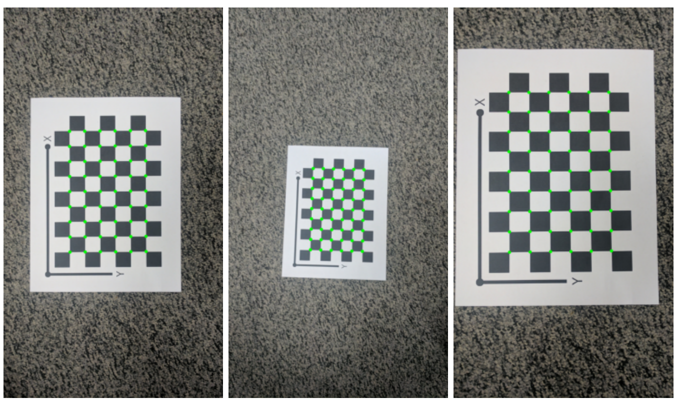


(a) Image - 1 (b) Image - 2 (c) Image - 3

Fig. 1: Initial Corners Detected

References

[1] Z. Zhang, "A flexible new technique for camera calibration," *IEEE Transactions on pattern analysis and machine intelligence*, vol. 22, no. 11, pp. 1330–1334, 2000.



(a) Image - 1 (b) Image - 2 (c) Image - 3

Fig. 2: Reprojected Corners