

# RBE/CS549: Project 4

## Deep and Un-Deep Visual Inertial Odometry

Prasanna Natu  
*M.S. Robotics Engineering*  
*Worcester Polytechnic Institute*  
 Worcester, MA  
 pvnatu@wpi.edu

Peter Dentch  
*M.S. Robotics Engineering*  
*Worcester Polytechnic Institute*  
 Worcester, MA  
 pdentch@wpi.edu

**Abstract**—This project aims to implement Vision-aided Odometry utilizing the Multi-state Constraint Kalman Filter (MSCKF) approach. The motivation behind adopting MSCKF for Visual Inertial Odometry lies in addressing the challenges faced by autonomous vehicles in employing high-quality sensors and efficient processors. By implementing this approach, it is possible to achieve accurate state estimation and localization without relying on costly and heavy sensor systems. In this project, a filter-based method is developed, fusing data from two sensors: a stereo camera and an IMU. The MSCKF is employed to estimate the state of the robot, while sensor fusion of the IMU and stereo camera is used to determine its localization

**Index Terms**—**R**Index Terms: Visual Inertial Odometry, Multi-state Constraint Kalman Filter, MSCKF, sensor fusion, stereo camera, inertial measurement unit, IMU.

### I. PHASE I

#### A. INITIALIZING GRAVITY AND BIAS

The 6-DOF IMU sensor utilized in this project is prone to biases that must be corrected in each reading. The 6-DOF represents three degrees of freedom for rotation (gyroscope) and three degrees of freedom for acceleration (accelerometer). This correction process can be considered as the calibration of the IMU sensor, where the biases in both rotation and acceleration are calculated and subsequently subtracted from every subsequent IMU reading.

This calibration is performed by keeping the rotor stationary and collecting approximately 100-200 readings, followed by calculating the mean of these readings. Ideally, the gyroscope reading should be [0, 0, 0]; however, due to noise and bias, small fluctuations in the gyroscope reading are observed.

To correct for these fluctuations, the mean of the collected readings is subtracted from the subsequent IMU readings while the rotor is stationary, as described earlier. Ideally, the accelerometer reading should be [0, 0, -g] in the world frame. Nevertheless, noise and biases in the low-cost IMU sensor cause fluctuations

The overview of the traditional method which we implemented is represented below:

$$\mathbf{x}_I = \left( \begin{matrix} {}^I\mathbf{q}^\top & \mathbf{b}_g^\top & {}^G\mathbf{v}_I^\top & \mathbf{b}_a^\top & {}^G\mathbf{p}_I^\top & {}^I\mathbf{c}\mathbf{q}^\top & {}^I\mathbf{p}_C^\top \end{matrix} \right)^\top \quad (1)$$

Fig. 1. State Vector

$$\tilde{\mathbf{X}}_k = \left[ \begin{matrix} \tilde{\mathbf{X}}_{IMU_k}^T & \delta\theta_{C_1}^T & {}^G\tilde{\mathbf{p}}_{C_1}^T & \dots & \delta\theta_{C_N}^T & \mathbf{p}_{C_N} \end{matrix} \right]^T \quad (2)$$

Fig. 2. Error State Vector

#### B. BATCH IMU PROCESSING

IMU batch processing is carried out to process IMU messages until the next set of images from the stereo camera becomes available. Prior to this, it is crucial to define the state vector to estimate subsequent states. The state vector comprises states in both the camera and IMU.

As depicted in Figure 1, the state vector includes the quaternion  $\mathbf{q}$ , which describes the rotation from the global to the IMU frame. The variables  $\mathbf{b}_g$  and  $\mathbf{b}_a$  represent the biases in the gyroscope and accelerometer, respectively. The position and velocity of the body frame in the inertial (world) frame are denoted by  $\mathbf{p}_I$  and  $\mathbf{v}_I$ . The transformations between the IMU and camera frames are represented by  $q_{Ic}$  and  $p_{Ic}$ .

For  $N$  camera poses, the state vector adds a new state in the buffer, with the first element being the states associated with the IMU sensor.

The objective of the batch IMU processing function is to predict the subsequent state and update the state information using the process model for a given time step, based on the IMU messages. The state information is updated after processing the IMU data and concludes after a specified time duration.

### C. Process Model

The process model predicts the IMU state using a motion model derived from error states, as shown in Figure 2. The error in quaternion is a quaternion operation, represented as:

$$\delta q = q \otimes q_0^{-1} \quad (3)$$

The  $\hat{\omega}$  and  $\hat{a}$  are given as follows:

$$\hat{\omega} = \omega_m - b_g \quad (4)$$

$$C^t \dot{\hat{q}} = \frac{1}{2} \Omega(\hat{\omega})^l_G \hat{q}, \quad \dot{b}_g = 0_{3 \times 1},$$

$$G_{\hat{v}}^l = C (G^l \hat{q})^\top \hat{a} + G g,$$

$$\dot{b}_a = 0_{3 \times 1}, \quad G_{\hat{p}j} = a \dot{v},$$

$$C \dot{q} q = 0_{3 \times 1}, \quad I \dot{p} C = 0_{3 \times 1}$$

Fig. 3. IMU dynamics

$$\Phi_k = \Phi(t_{k+1}, t_k) = \exp\left(\int_{t_k}^{t_{k+1}} F(\tau) d\tau\right)$$

$$Q_k = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) G Q G^\top \Phi(t_{k+1}, \tau)^\top d\tau$$

Fig. 4. Covariance Matrix

$$\hat{a} = a_m - b_g \quad (5)$$

The other errors are additive errors that simply add to the previous quantity. These error states are employed to determine the robot's process model. Angular velocity and linear acceleration are derived, as illustrated in Figure 3, where  $\Omega$  is the quaternion derivative and is expressed as:

$$\Omega(\omega) = \begin{bmatrix} \omega & \hat{\omega} \\ \omega^T & 0 \end{bmatrix} \quad (6)$$

Here,  $\hat{\omega}$  is a skew-symmetric matrix of the  $\omega$  vector. The linearized continuous error dynamics of the IMU error state are defined as follows:

$$\dot{\tilde{X}}_I = F \tilde{X}_I + G n_I \quad (7)$$

The term  $n_I$  denotes the Gaussian noise of the accelerometer and gyro readings. To propagate the IMU measurement in discrete time, the 4th-order Runge Kutta method is applied.

The F matrix in the above equation (discrete-time equation) is utilized to derive the discrete-time state transition matrix, and the G matrix is employed to obtain the discrete-time noise covariance matrix.  $\phi_K$  is approximated using a Taylor expansion up to the 3rd order of F, while  $Q_k$  is a discrete-time state covariance matrix obtained by continuous-time methods of state covariance Q and G matrix. The observability constraint is applied by modifying the transition matrix. The state transition matrix is corrected by making it symmetric in this step.

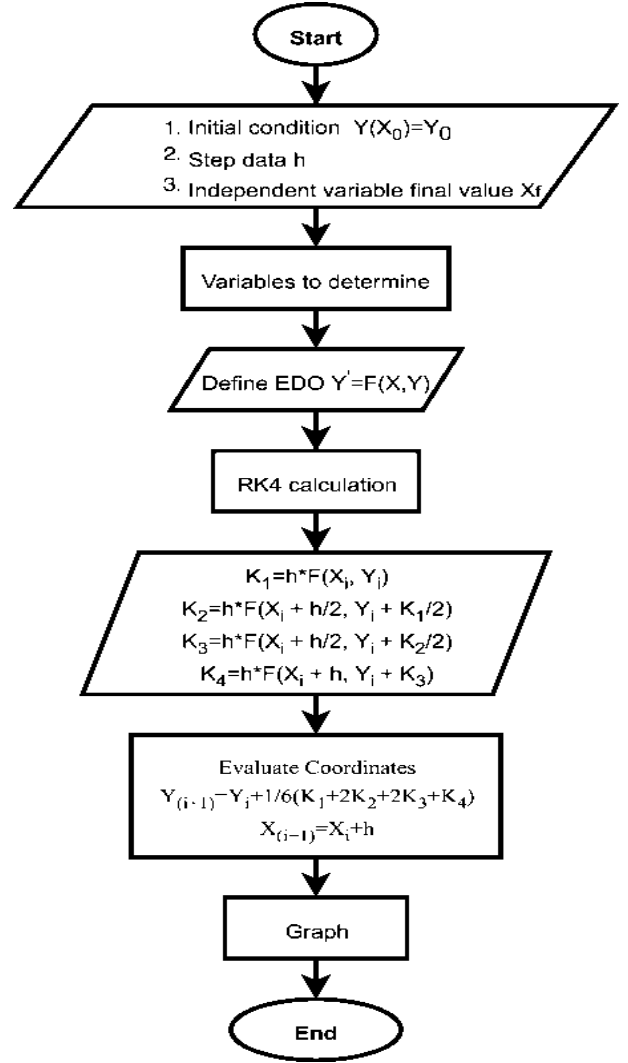


Fig. 1. Runge Kutta

### D. State Augmentation

When new images are received, the state should be augmented with the new camera state. The pose of the new camera state can be computed from the latest IMU state.

The augmented covariance matrix is given by the following equation in Section VI, and the J matrix is given by the equation in Section VI.

### E. Adding Feature Observation

Here, we check if the feature observed is in the map server dictionary or not. If it is not there, we add a new key to the map server dictionary.

### F. Measurement Model and Update

A single feature  $f_j$  is observed by the stereo cameras with the pose. The stereo cameras have different poses, for left and right cameras respectively, at the same time instance. Although the state vector only contains the pose of the left camera,

the pose of the right camera can be easily obtained using the extrinsic parameters from the calibration.

The dimension is then reduced to  $\mathbb{R}^3$  assuming the stereo images are properly rectified. However, by representing the same in  $\mathbb{R}^4$ , we can skip the rectification, and the camera poses are given by:

The position of the feature in the world frame is calculated using Gaussian-Newton least square minimization. The residual of measurement can be approximated by the following equation:

### G. State Augmentation

The global frame feature pose is determined using the camera pose, which causes the uncertainty of  $p_j$  in the global frame to be related to the camera states. By projecting the residual in equation (4) onto the null space  $V$  of  $HJ$ , this correlation is removed.

- 1) Determine Cam0 pose.
- 2) Determine Cam1 pose.
- 3) Identify 3D feature position in the world frame and its observation using stereo cameras.
- 4) Transform the feature position from the world frame to the cam0 and cam1 frame.
- 5) Adjust the measurement Jacobian to maintain observability constraint.
- 6) Calculate the residual.

## II. UPDATING PROCEDURE

The updating process is executed in the following manner:

- 1) Verify if  $H$  and  $r$  are empty.
- 2) Perform decomposition on the final Jacobian matrix to minimize computational complexity.
- 3) Determine the Kalman gain.
- 4) Calculate the state error.
- 5) Update the IMU state.
- 6) Modify the camera states.
- 7) Refresh state covariance.
- 8) Ensure covariance symmetry.

## III. RESULTS

The outcomes of our implementation are presented below.

### REFERENCES

- [1] RBE 549: Robot Perception (2023). Project 4: Visual Inertial Odometry. Retrieved from <https://rbe549.github.io/spring2023/proj/p4/>
- [2] Sun, K., Mohta, K., Pfrommer, B., Watterson, M., Liu, S., Mulgaonkar, Y., Taylor, C. J., Kumar, V. (YEAR). Robust Stereo Visual Inertial Odometry for Fast Autonomous Flight.
- [3] Mourikis, A. I., Roumeliotis, S. I. (YEAR). A Multi-State Constraint Kalman Filter for Vision-aided Inertial Navigation.

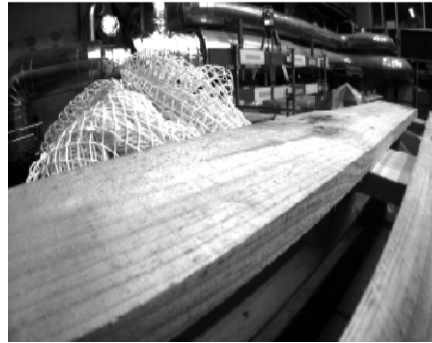
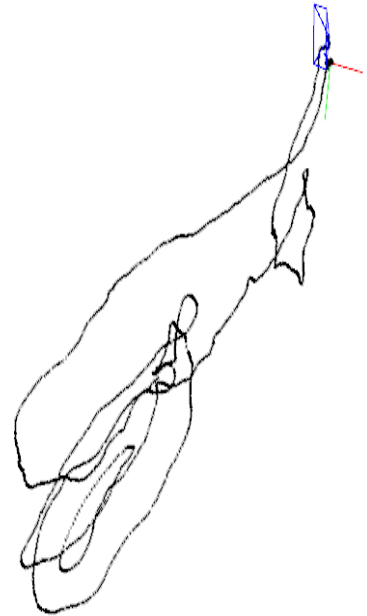


Fig. 2. Output