Homework 1: AutoCalib

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1. Introduction

Zhang's camera calibration is one of the most used camera calibration techniques in computer vision. It is used for estimating the intrinsic parameters of a camera like focal length, distortion, and principle point, unlike the DLT method, which estimates both the intrinsic and extrinsic parameters like camera pose. Thirteen checkerboard pattern images were used for calibration. Size of each square was 21.5mm.

$$K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 1. Camera Caliberation Matrix

2. Image Points and World Points

The image points were identified and stored for all the 13 images, using cv2.findChessboardCorners.The corners are obtained row by row, left to right.

The world points were generated in a numpy grid. The world points were generated by multiplying the the size of square, 21.5 mm, based on the location of the corner from the top left corner.

3. Initial Parameter Estimation

3.1. Estimating H matrix

The H matrix was estimated first based on the method given under Section 3.1 [1]. For each image, the x and y coordinates of the corner image points and world points are separated to form 2 expressions per point. After getting all the equations, they are solved using SVD, and we receive the H matrix. So, for 13 images, we get 13 H matrices.

3.2. Estimating K matrix

The relation between the image and world points can be established by x=KRX, where K is the camera intrinsic matrix, R is the matrix having the pose of the camera, and X is the world points. x is the image points. Based on the method given in Section 3.1 [1], for each H matrix, each column is split, and then formulate the expressions for V. From Equation in Fig. 2, we find out b matrix, based on formulation for V. Then, using the b = [B11, B12, B22, B13, B23, B33].T, we find out the K matrix, using relation between elements of b, and K. Also, the radial distortion matrix, k =[[k1],[k2]] was estimated as 0.The estimated B and K matrix are included in the Fig. 3 and Fig. 4.

```
[[-1.50329250e-07 -1.97319403e-10  1.14760224e-04]
 [-1.97319403e-10 -1.52265491e-07  2.07961474e-04]
 [ 1.14760224e-04  2.07961474e-04 -9.99999972e-01]]
```

Figure 2. B matrix

```
[[ 2.04515122e+03 -2.66730758e+00  7.61601108e+02]
 [ 0.00000000e+00  2.03210805e+03  1.36479509e+03]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```

Figure 3. Estimated K matrix

3.3. Estimating the Camera Pose

The pose of the camera is then calculated, after the estimation of K. It is done using the equations which are given below. The pose consists of r1,r2,r3 rotations and t translation vector.

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$$
\begin{aligned}
\mathbf{r}_1 &= \lambda \mathbf{A}^{-1}\mathbf{h}_1 \\
\mathbf{r}_2 &= \lambda \mathbf{A}^{-1}\mathbf{h}_2 \\
\mathbf{r}_3 &= \mathbf{r}_1 \times \mathbf{r}_2 \\
\mathbf{t} &= \lambda \mathbf{A}^{-1}\mathbf{h}_3
\end{aligned}
$$

with $\lambda = 1/\|\mathbf{A}^{-1}\mathbf{h}_1\| = 1/\|\mathbf{A}^{-1}\mathbf{h}_2\|$

Figure 4. K matrix to RT Matrix

4. Non-Linear Geometric Error Minimization

$$\sum_{i=1}^{N}\sum_{j=1}^{M} \left|\left| x_{i,j} - \hat{x}_{i,j}\left(K, R_i, t_i, X_j, k\right) \right|\right|$$

Figure 5. Loss function

Once we give all the initial estimates for the variables, we find out the re-projection error based on the equation in Fig. 5. We minimize the loss by using scipy.optimize. The first term in the loss function is image point, and the second term is the projection.

5. Results

The re-projection error before optimization was 0.7646751902747165 and after optimization, it reduced to 0.7518997828266477. Also, the K matrix and the radial distortion vector received after optimization are given in the image below.

```
[[ 2.04514716e+03 -2.66769110e+00  7.61606510e+02]
 [ 0.00000000e+00  2.03210074e+03  1.36480822e+03]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
[[ 0.01191912]
 [-0.08208295]]
```

Figure 6. K (Intrinsic Parameters) and k (Radial Distortion Parameters)

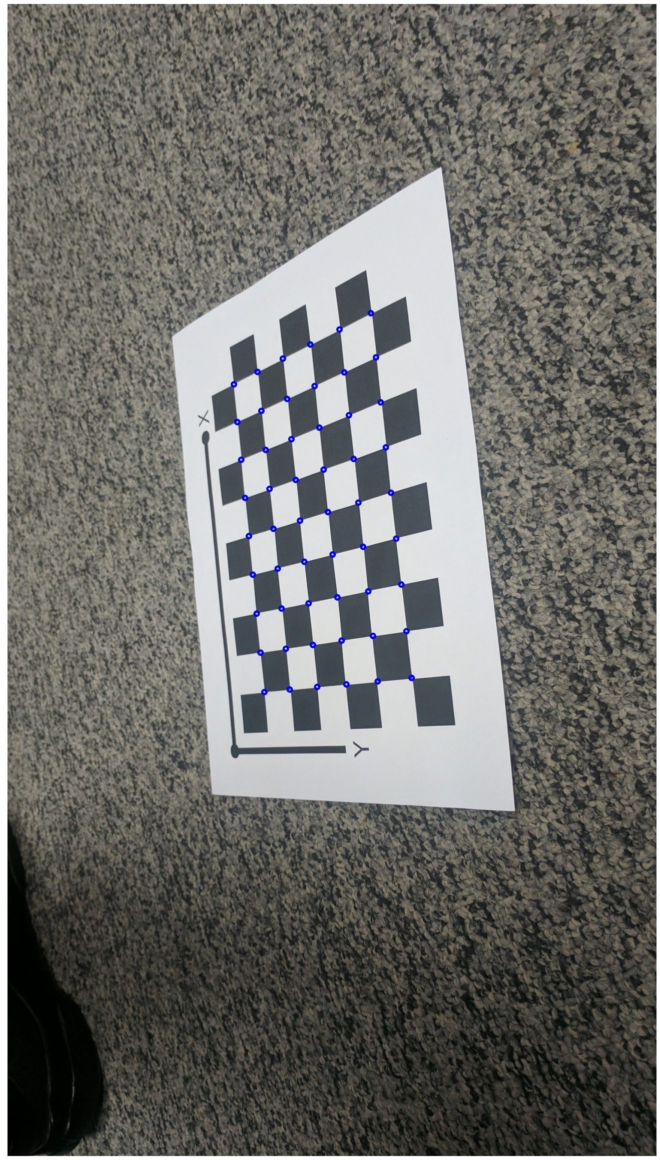

Figure 7. Reprojection of corners in image 1

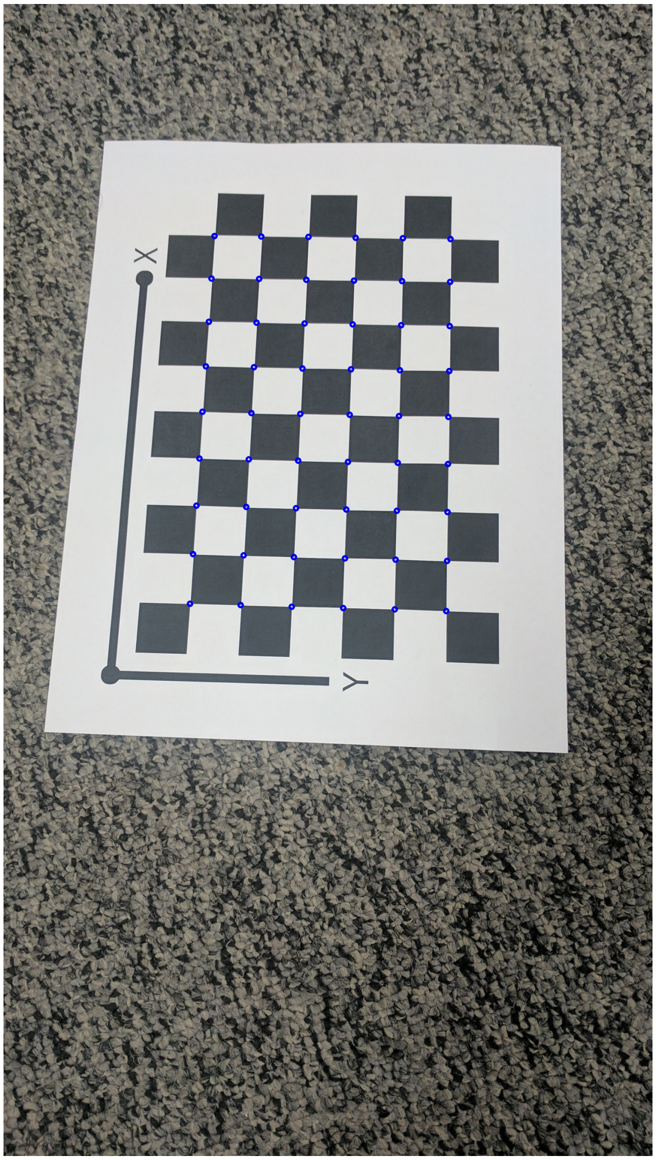

Figure 8. Reprojection of corners in image 2

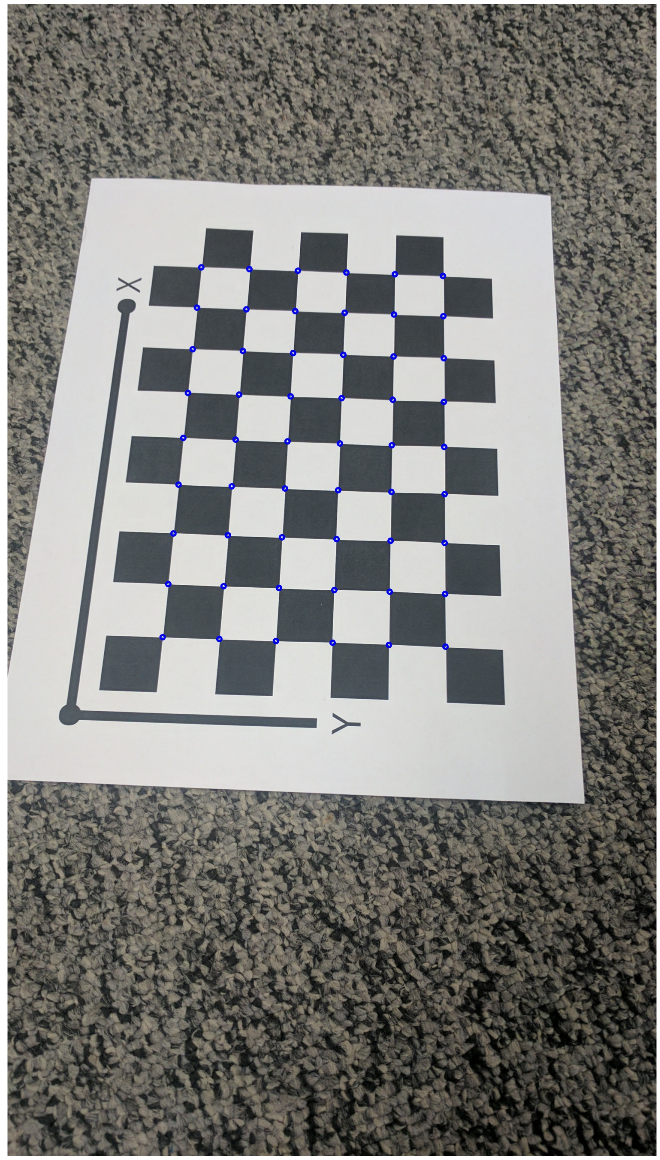

Figure 9. Reprojection of corners in image 3

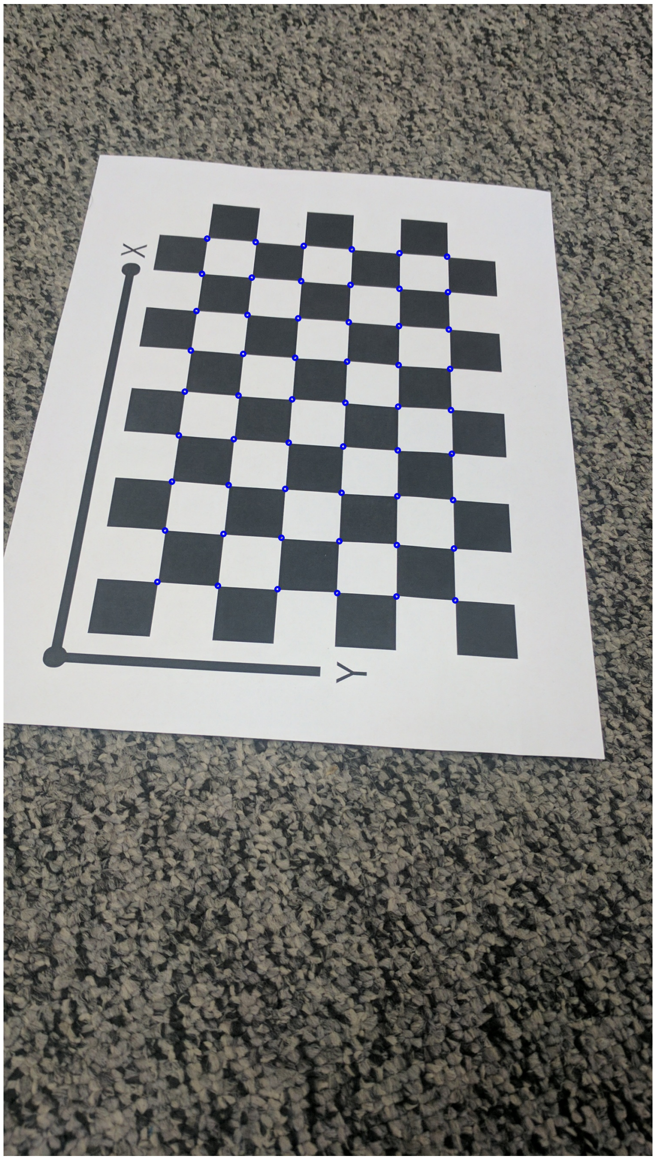

Figure 10. Reprojection of corners in image 4

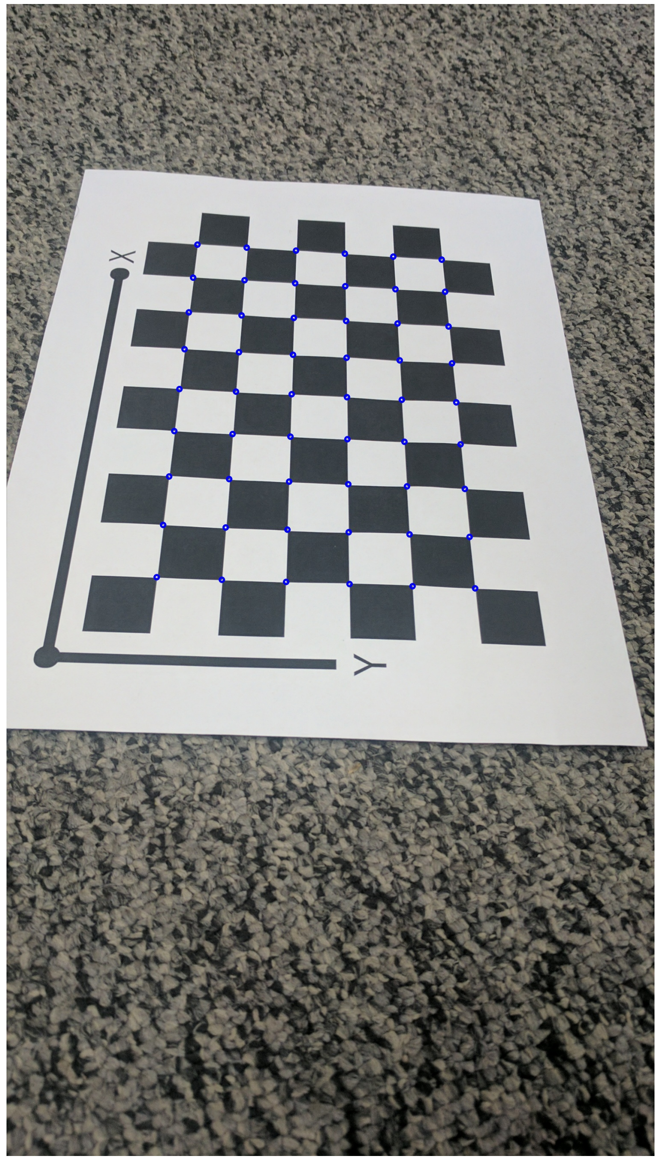

Figure 11. Reprojection of corners in image 5

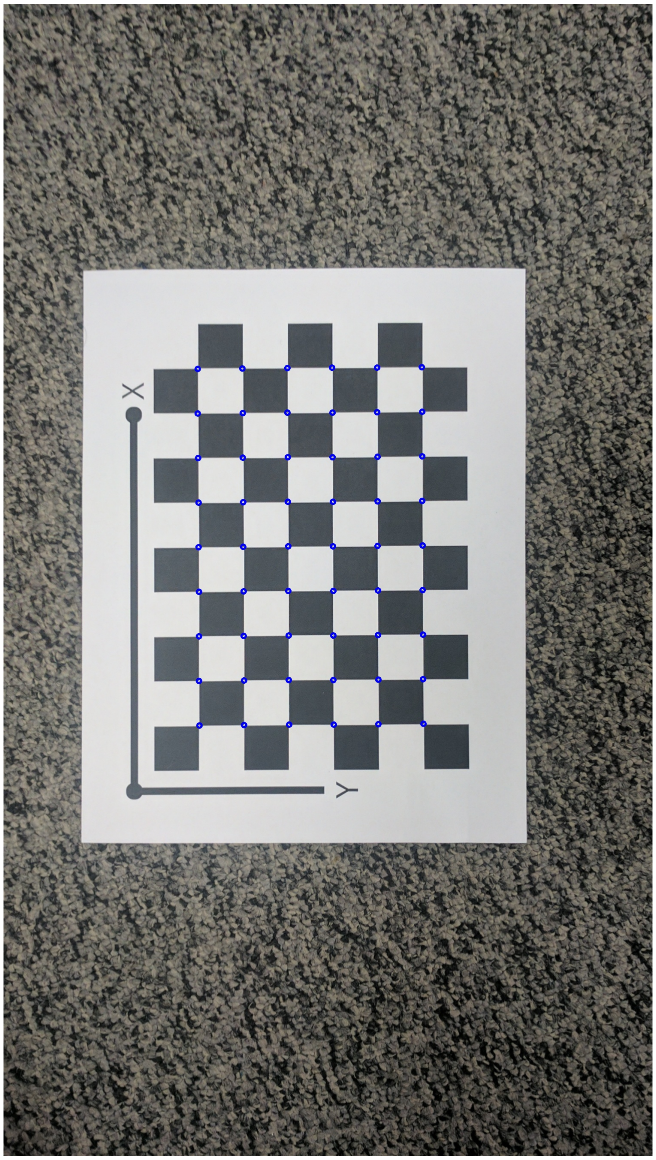

Figure 12. Reprojection of corners in image 6

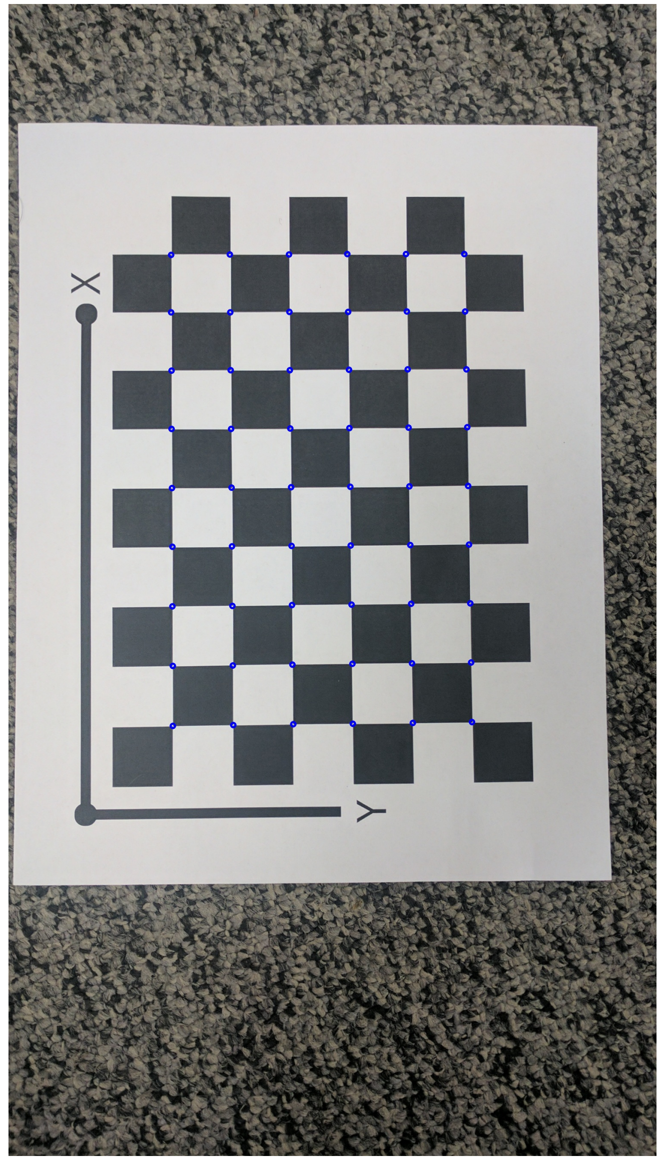

Figure 13. Reprojection of corners in image 7

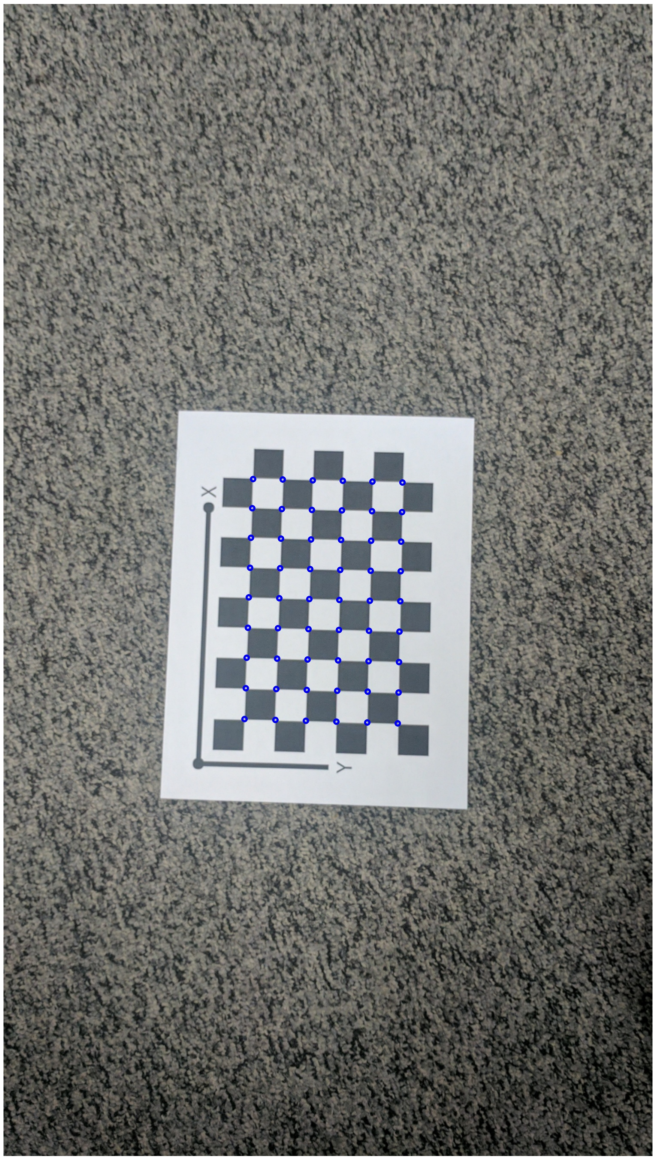

Figure 14. Reprojection of corners in image 8

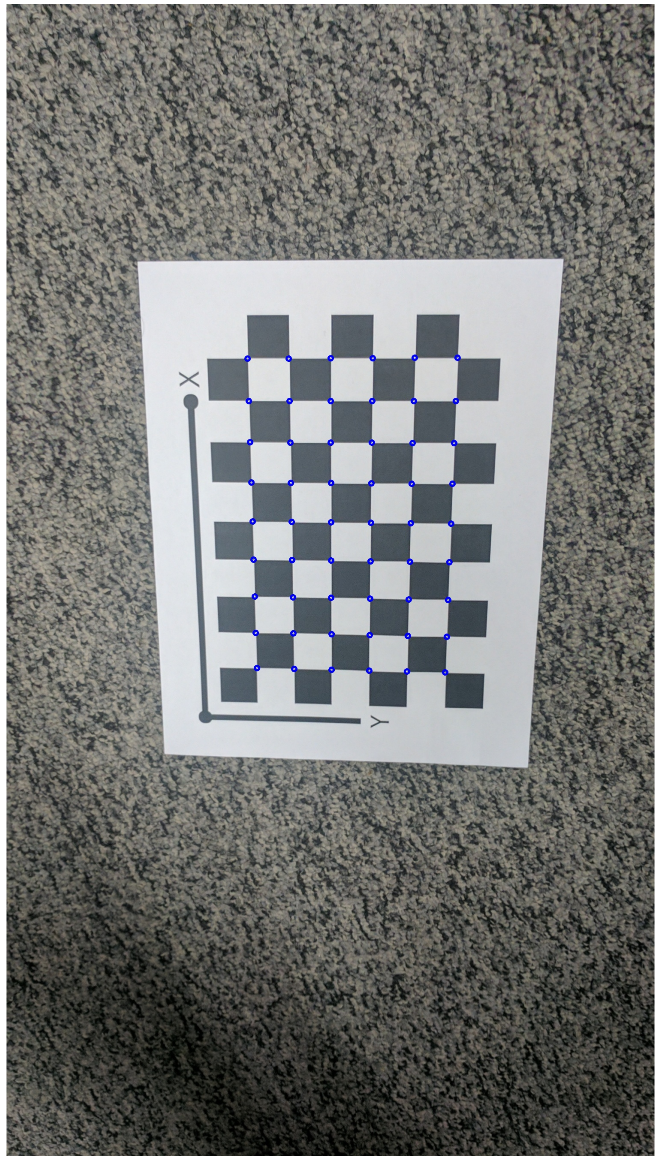

Figure 15. Reprojection of corners in image 9

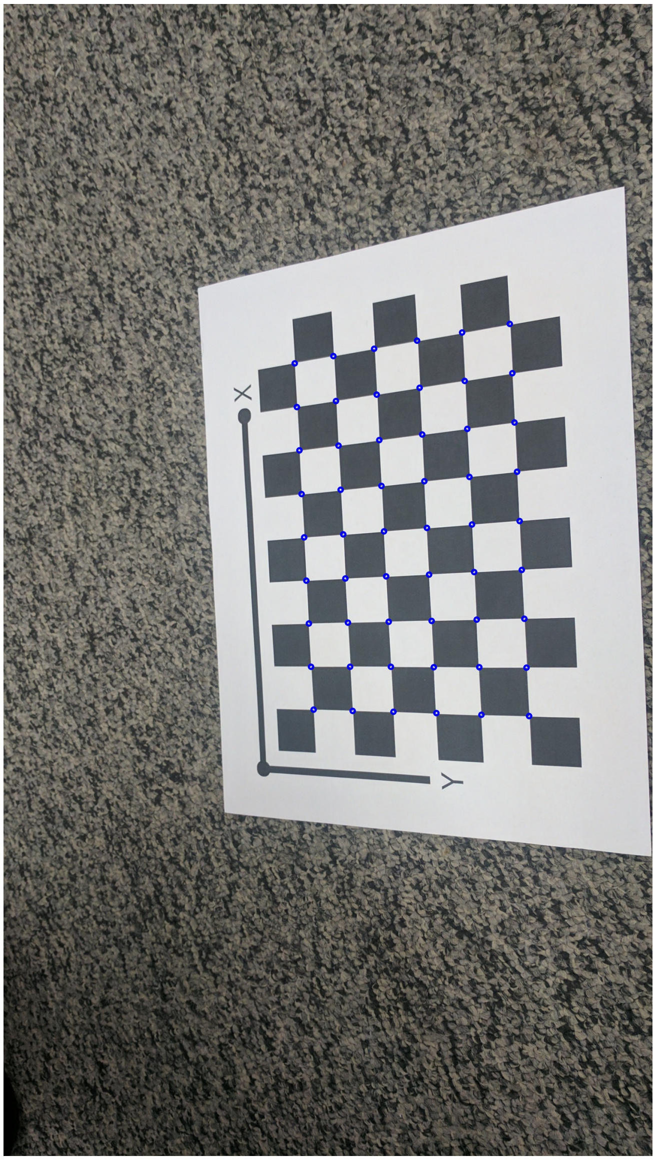

Figure 16. Reprojection of corners in image 10

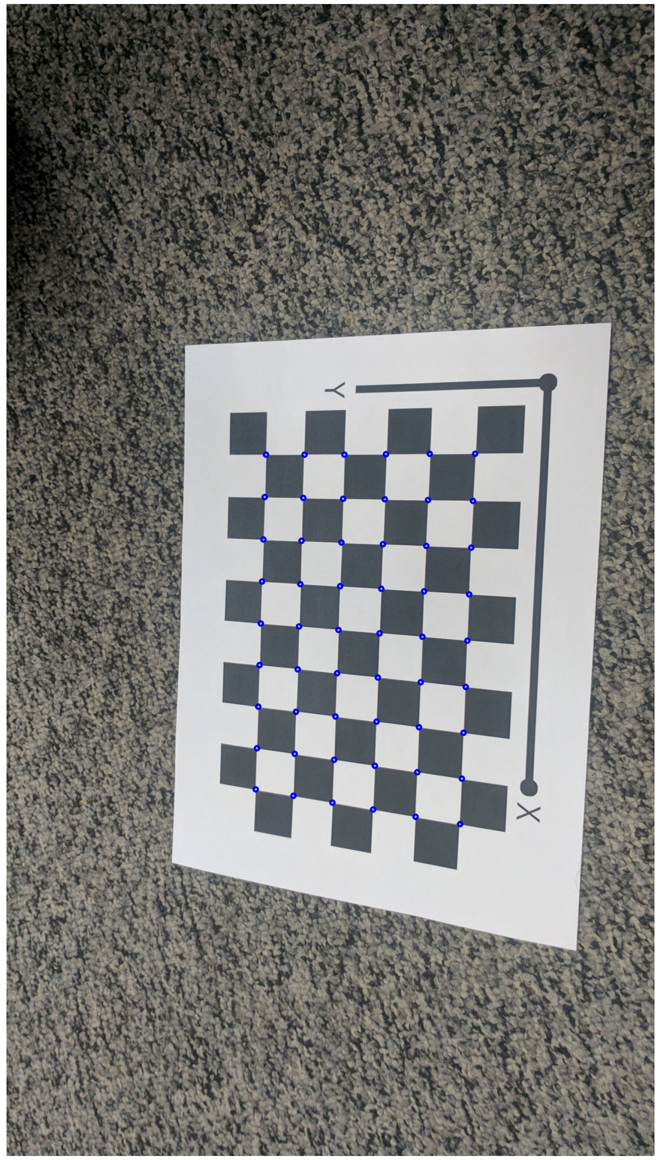

Figure 17. Reprojection of corners in image 11

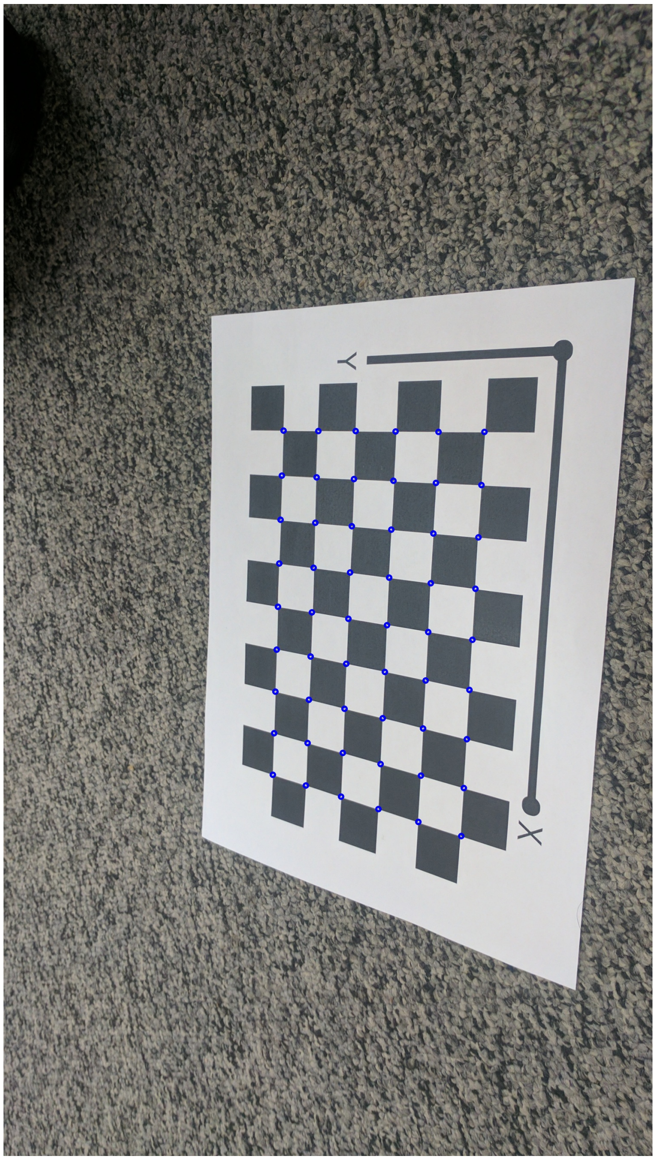

Figure 18. Reprojection of corners in image 12

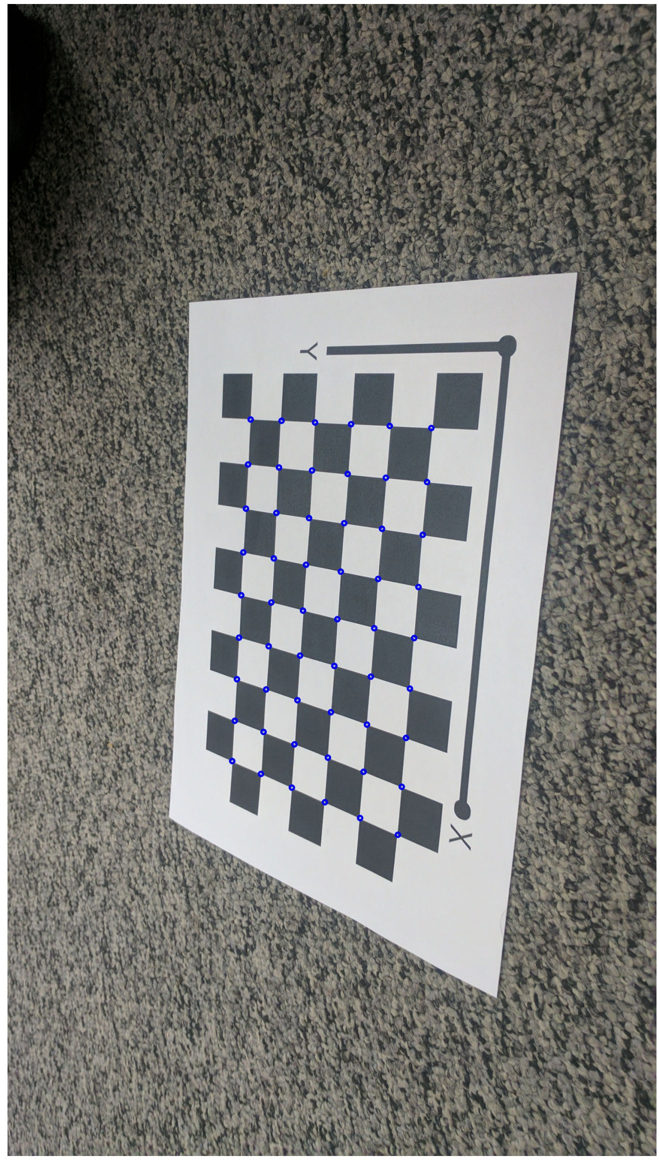

Figure 19. Reprojection of corners in image 13