RBE/CS549: Computer Vision Homework 1 - AutoCalib

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Using 1 Late Day

***Abstract*—Camera calibration involves determining the various parameters associated with a camera, such as its focal length, distortion coefficients, and the principle point. In this homework, we will be implementing "A Flexible New Technique for Camera Calibration" proposed by Zhang.**

Index: Camera calibration, Intrinsic parameters, Extrinsic parameters, Distortion

A. Overview

In this homework, a checkerboard pattern with each square size of 21.5 mm has been given using which camera has to be calibrated. To convert a real-world point to its corresponding representation in an image, there are two types of transformation matrices involved. First, the camera calibration matrix with the coordinates of the principle point, the scale factors in image axes and second, with extrinsic parameters, the rotation and translation which relates the world coordinate system to the camera coordinate system. Further, with this, a nonlinear optimization is computed using initial estimates to obtain minimum projection error and define the intrinsic matrix parameters and the distortion coefficients.

B. Estimation of intrinsic parameter matrix

The camera intrinsic matrix consists of (u_0, v_0) the coordinates of the principal point, α and β the scale factors in image u and v axes, and γ the parameter describing the skewness of the two image axes.

$$\mathbf{A} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

For estimating homography between the model plane and its image, first the pixel coordinates of the chessboard corners are detected using the cv2.findChessboardCorners function. Then calculated the real-world co-ordinates of the chessboard with the help of size of the square. Using these coordinates, Homography matrix using cv2.findHomography function. This homography matrix consists of both intrinsic matrix and extrinsic matrix. Assuming the model plane is on Z = 0 of the world coordinate system, we get following relation.

As we know, r1 and r2 are orthonormal, following two equations can be obtained:

Further using Cholesky Decomposition, following closed-form solution is computed:

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} .$$

$$\mathbf{H} = \mathbf{A} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 = 0$$
$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2$$

$$\mathbf{B} = \mathbf{A}^{-T}\mathbf{A}^{-1} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}$$

Here, as B is symmetric, a 6D vector is defined,

$$\mathbf{b} = [B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]^T$$

Hence, from homography, homogeneous equations can be written as:

$$\mathbf{v}_{ij} = [h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3}]^T$$

$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = \mathbf{0}$$

From above, we can calculate b vector and using this vector intrinsic parameters can be calculated as follows:

$$\begin{aligned} Vb &= 0 \\ v_0 &= (B_{12}B_{13} - B_{11}B_{23})/(B_{11}B_{22} - B_{12}^2) \\ \lambda &= B_{33} - [B_{13}^2 + v_0(B_{12}B_{13} - B_{11}B_{23})]/B_{11} \\ \alpha &= \sqrt{\lambda/B_{11}} \\ \beta &= \sqrt{\lambda B_{11}/(B_{11}B_{22} - B_{12}^2)} \\ \gamma &= -B_{12}\alpha^2\beta/\lambda \\ u_0 &= \gamma v_0/\beta - B_{13}\alpha^2/\lambda \end{aligned}$$

C. Computation of extrinsic parameters

As intrinsic parameter matrix is now known, the extrinsic parameters for each image can be computed:

$$\begin{aligned} r_1 &= \lambda A^{-1} h_1 \\ r_2 &= \lambda A^{-1} h_2 \\ r_3 &= r_1 \times r_2 \\ t &= \lambda A^{-1} h_3 \end{aligned}$$

Therefore, with these we have intrinsic as well as extrinsic parameters.

D. Estimation of Distortion and Projection error

With the assumption to have minimum camera distortion, distortion coefficients are considered as $k = [0,0]^T$ initially. With the initial estimates, we try minimization of the re-projection error using the equation.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \|\mathbf{m}_{ij} - \hat{\mathbf{m}}(\mathbf{A}, \mathbf{R}_i, \mathbf{t}_i, \mathtt{M}_j)\|^2$$

$$\begin{aligned} \breve{u} &= u + (u - u_0)[k_1(x^2+y^2) + k_2(x^2+y^2)^2] \\ \breve{v} &= v + (v - v_0)[k_1(x^2+y^2) + k_2(x^2+y^2)^2] \end{aligned}$$

Here, (u, v) are the ideal (nonobservable distortion-free) pixel image coordinates, $(\breve{u}, \breve{v})$ are the corresponding real observed image coordinates and k_1 and k_2 are the coefficients of the radial distortion. Further Least square function takes this error to optimize the parameters. With the help of the optimized parameters, we calculate mean of reprojected error and the points.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \|\mathbf{m}_{ij} - \breve{\mathbf{m}}(\mathbf{A}, k_1, k_2, \mathbf{R}_i, \mathbf{t}_i, \mathtt{M}_j)\|^2$$

E. Results

Calibration matrix before and after optimization is as below:

$$\mathbf{A} = \begin{bmatrix} 2065.25652 & -2.93974703 & 764.676160 \\ 0.00000000 & 2053.48352 & 1362.76925 \\ 0.00000000 & 0.00000000 & 1.00000000 \end{bmatrix}$$

$$\mathbf{A_{opt}} = \begin{bmatrix} 2065.25110 & -2.94103239 & 764.677758 \\ 0.00000000 & 2053.48052 & 1362.81138 \\ 0.00000000 & 0.00000000 & 1.00000000 \end{bmatrix}$$

The distortion coefficients obtained after optimization are:

$$\mathbf{K_{opt}} = \begin{bmatrix} 0.01652818 & -0.11331139 \end{bmatrix}$$

The value of mean projection error after optimization is 0.7885893658.

The detected corners for given images are illustrated in Fig.01 to Fig.05, whereas the re-projected points for corresponding images are illustrated in the Fig.06 to Fig.10. Also projection error before optimization and after optimization is tabulated for these 5 images.

Error before Optimization	Error after Optimization
0.69110220	0.63863823
0.75187334	0.72412466
0.66584360	0.62158755
0.65136636	0.64343234
1.02835226	0.97809391

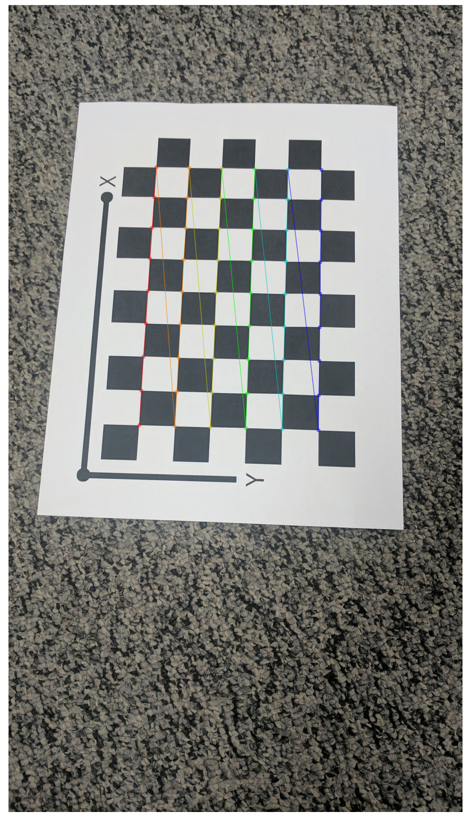

Fig. 1. Detected corners for Image:1

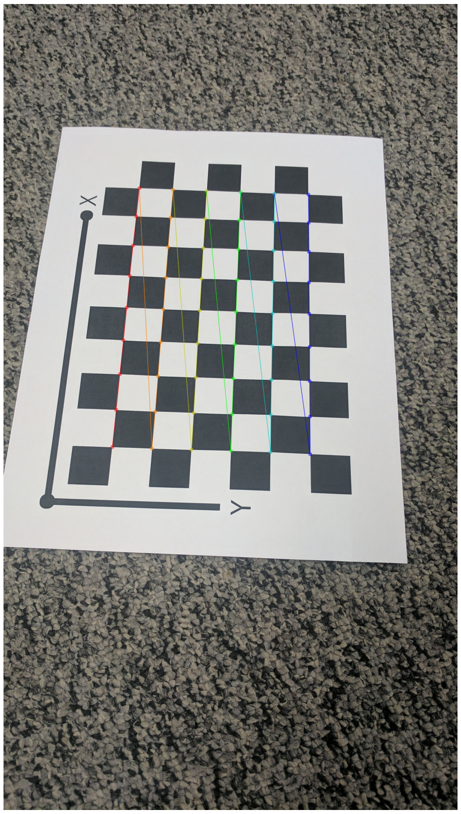

Fig. 2. Detected corners for Image:2

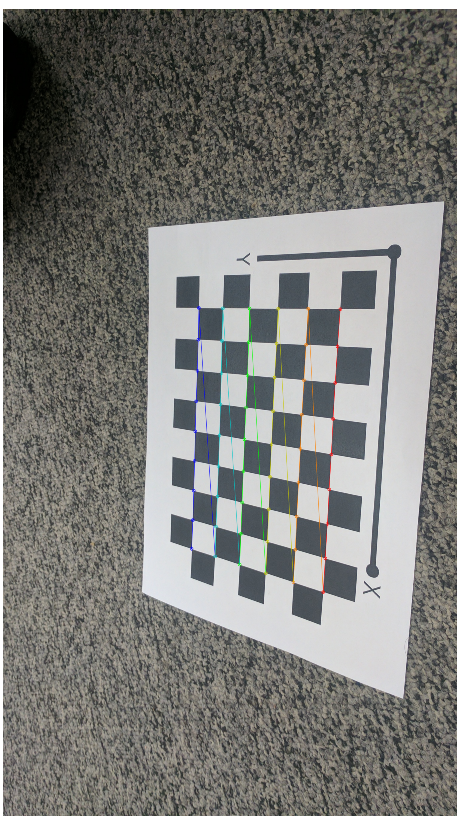

Fig. 3. Detected corners for Image:3

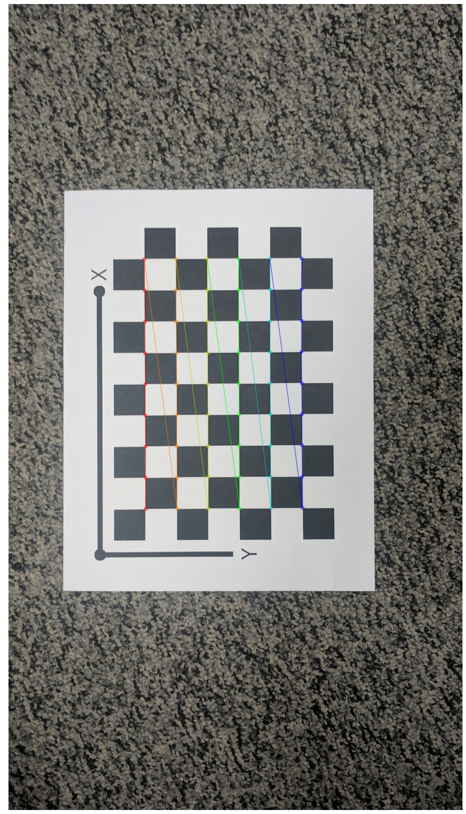

Fig. 4. Detected corners for Image:4

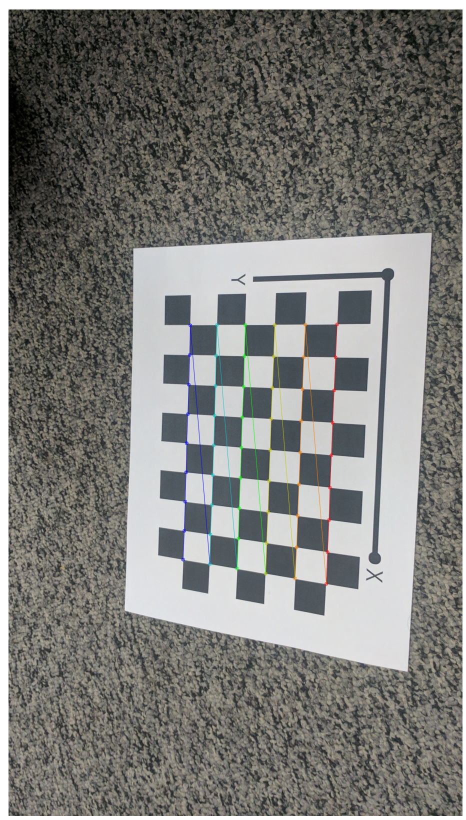

Fig. 5. Detected corners for Image:5

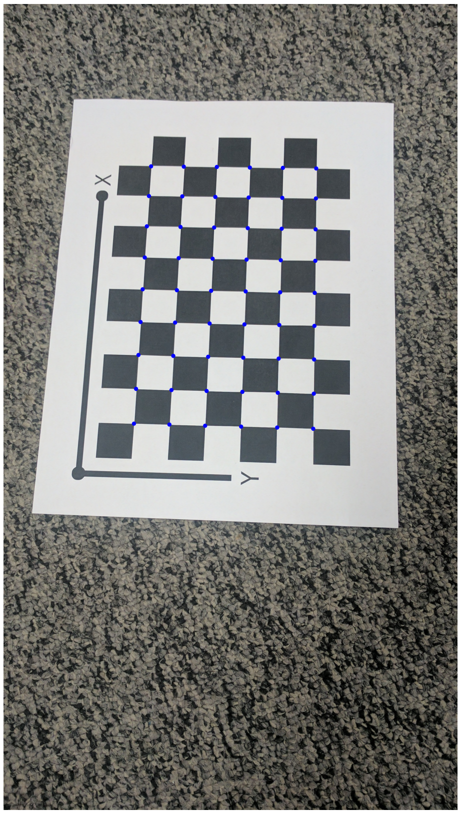

Fig. 6. Re-projected corners on Image:1

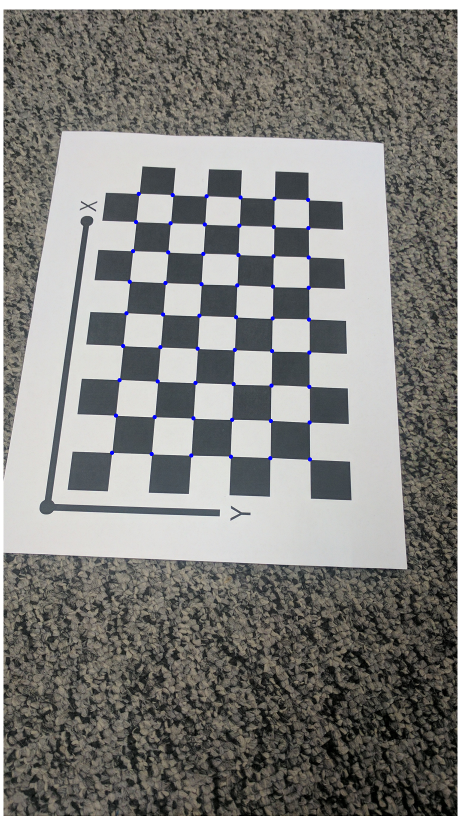

Fig. 7. Re-projected corners on Image:2

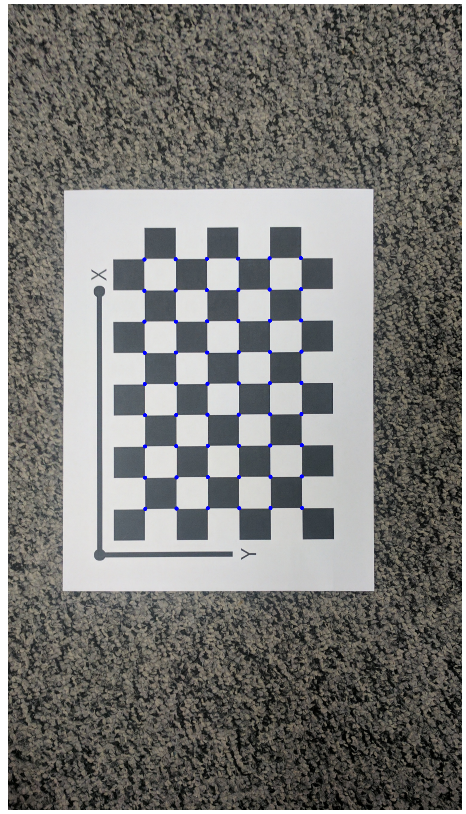

Fig. 8. Re-projected corners on Image:3

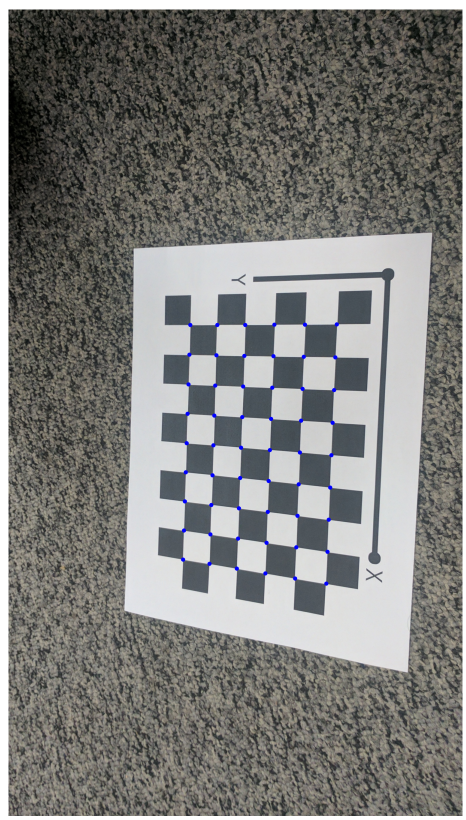

Fig. 9. Re-projected corners on Image:4

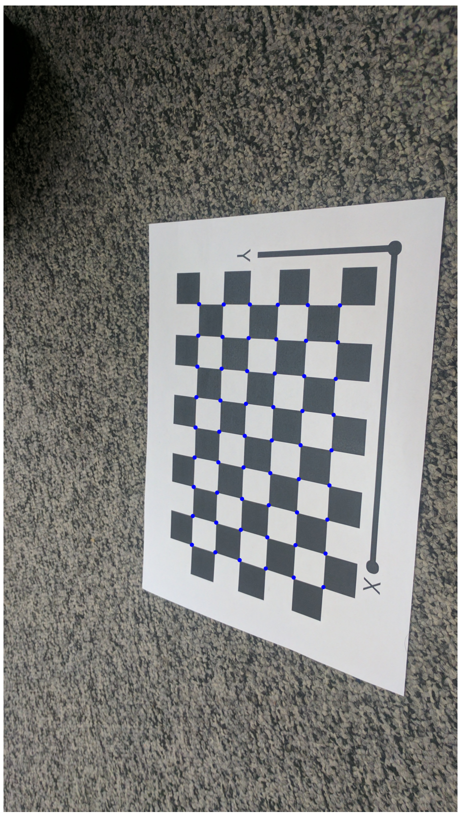

Fig. 10. Re-projected corners on Image:5